

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/AS55

B.SC. DEGREE EXAMINATION, Dec 2020

BRANCH I – MATHEMATICS

FIFTH SEMESTER

COURSE : MAJOR CORE

PAPER : **ALGEBRAIC STRUCTURES**

TIME : 90 minutes

MAXIMUM MARKS : 50

SECTION –A

Answer **ALL** the questions ($3 \times 2 = 6$)

1. Show that the order of an element of a finite group divides the order of the group.
2. Find the inverse of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$.
3. Prove that if U and V are ideals of a ring R , then $U + V$ is also an ideal of R .

SECTION –B

Answer **ANY THREE** questions ($3 \times 8 = 24$)

4. Show that the set of all 2×2 matrices with real entries and determinant 1 forms a group under matrix multiplication.
5. State and prove Lagrange's theorem.
6. Prove that the set of all even permutations A_n in S_n form a normal subgroup of index 2 in S_n .
7. Let R be the ring of integers and let U be the set of all integers consisting of the multiples of 17. Prove that U is a maximal ideal of R .

SECTION –C

Answer **ANY ONE** question ($1 \times 20 = 20$)

8. (a) Let G be a group and H a subgroup of G . Prove that the relation $a \equiv b \pmod{H}$ is an equivalence relation.
(b). Prove that if H and K are finite subgroups of G , then $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$. (8+12)
9. Prove that every integral domain can be imbedded in a field.
