STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/PS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2020
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : PROBABILITY AND STOCHASTIC PROCESSES
TIME : 1 ½ HOURS **MAX. MARKS : 50**

SECTION – A

ANSWER ALL THE QUESTIONS (2× 2 = 4)

1. Define a counting process.
2. Explain Stopping time.

SECTION – B

ANSWER ANY TWO QUESTIONS (2 × 6 = 12)

3. State and prove Borel- Cantelli Lemma.
4. Suppose that travelers arrive at a railway station in accordance with a Poisson Process with rate λ. If the train departs at time *t*, compute the expected sum of the waiting times of travelers arriving in (0, *t*).
5. Prove that in Markov Chains state *j* is recurrent if and only if $\sum_{n=1}^{\infty} P_{jj}^{n} = \infty$.

SECTION – C

ANSWER ANY TWO QUESTIONS (2 × 17 = 34)

6. a) Find the expectation and Variance of the sum of a random number of random variables.

   b) If $\{E_n, n \geq 1\}$ is either an increasing or decreasing sequence of events then prove that $\lim_{n\to\infty} P(E_n) = P(\lim_{n\to\infty} E_n)$. (8+9)

7. a) Explain the *M*/G/1 Queue in detail.

   b) Derive the Kolmogorov's backward differential Equations. (10+10)

8. a) If $N_i(t)$ represents the number of type-i events that occur by time *i*, *i*= 1,2, then prove that $N_1(t)$ and $N_2(t)$ are independent Poisson random variables.

   b) Prove that if *N* is a random time for the martingale $\{Z_n\}$, then the stopped process $\{\overline{Z_n}\}$ is also a martingale. (8+12)