STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086

(For candidates admitted from the academic year 2019 – 20)

SUBJECT CODE: 19MT/PC/PD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2020

BRANCH I - MATHEMATICS

THIRD SEMESTER

COURSE : CORE

PAPER : PARTIAL DIFFERENTIAL EQUATIONS

TIME : 1 ½ HOURS **MAX. MARKS : 50**

SECTION – A

ANSWER ALL THE QUESTIONS (2 × 2 = 4)

1. When can you say two partial differential equations $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$ are compatible?

2. Define the Dirac delta function.

SECTION – B

ANSWER ANY TWO QUESTIONS (2 × 6 = 12)

3. If a characteristic strip contains at least one integral element of $F(x,y,z,p,q) = 0$, then prove that it is an integral strip of the equation $F\left(x,y,z,z_x,z_y\right) = 0$.

4. Explain briefly about boundary conditions and its three types.

5. Obtain the solution of the wave equation $u_{tt=} c^2 u_{xx}$ under the conditions (i) $u(0,t) = u(2,t) = 0$ (ii) $u(x,0) = sin^3 \frac{\pi x}{2}$ (iii) $u_t(x,0) = 0$.

SECTION – C

ANSWER ANY TWO QUESTIONS (2 × 17 = 34)

6. a) Find the integral surface of the linear partial differential equation $xp + yq = z$ which contains the circle defined by $x^2 + y^2 + z^2 = 4$ and $x + y + z = 2$. (9)

   b) State and solve the Neumann problem for a rectangle. (8)

7. a) Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$. (9)

   b) Derive three dimensional Laplace's equation in cartesian coordinates. (8)

8.a) Solve the one-dimensional diffusion equation in the region $0 \le x \le \pi$, $t \ge 0$ subject to the conditions (i) T remains finite as $t \to \infty$ (ii) $T = 0$, if $x = 0$ and $x = \pi$ for all t

(iii) At $t = 0, T = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$ (8)

b) Derive one dimensional wave equation. (9)