STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 and thereafter)

SUBJECT CODE: 19MT/PC/FA34

M.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2020

COURSE: CORE **TIME: 90 MINUTES**
PAPER: FUNCTIONAL ANALYSIS **MAX.MARKS: 50**

Section A
Answer all questions (2 × 2 =4)

1. Define a Schauder basis and give an example.
2. Prove that $\|A^*\| = \|A\|$ where $A \in BL(H)$ and H is a Hilbert space.

Section B
Answer any two questions (2 × 6 = 12)

3. Prove that a linear map on a linear space X may be continuous with respect to some norm on X, but discontinuous with respect to another norm on X. Illustrate with an example.
4. Prove: Let X be a normed space and E be a subset of X. Then E is bounded in X if and only if $f(E)$ is bounded in K for every $f \in X'$.
5. Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Then show that the following conditions are equivalent.
   a) span $\{u_\alpha\}$ is dense in H.
   b) If $x \in H$ and $\langle x, u_\alpha \rangle = 0$ for all α, then $x = 0$.

Section C
Answer any two questions (2 × 17 = 34)

6. (a) Let X and Y be Banach spaces and $F: X \to Y$ be a closed linear map. Then prove that F is continuous.
   (b) If X and Y are normed spaces and $X \neq 0$. Then prove that $BL(X, Y)$ is a Banach space in the operator norm if and only if Y is a Banach space.
   (9+8)
7. (a) Let X be a separable normed space. Prove that every bounded sequence in X' has a weak* convergent subsequence.
   (b) State and prove Riesz representation theorem.
   (7+10)
8. (a) Let H be a Hilbert space and $A \in BL(H)$. Then prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$.
   (b) State and prove Generalized Schwarz inequality.
   (c) Prove that in a Hilbert space H, if $A, B \in BL(H)$ are normal and if A commutes with B^* and B commutes with A^*, then $A+B$ and AB are normal.
   (6+7+4)