STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)
B.SC. DEGREE EXAMINATION, April 2021
BRANCH I – MATHEMATICS

SUBJECT CODE: 19MT/MC/IC23

PAPER: Integral Calculus
TIME : 90 minutes **MAX. MARKS: 50**

Section – A

Answer *all* questions **(3 × 2 = 6)**

1. Figure out the infinite discontinuity of the function in the integral $\int_{-\infty}^{-3} \frac{dt}{t^3-7t+6}$.
2. Write any two properties of beta function.
3. Find the Jacobian of the transformation $x = u^2 - v^2, y = u^2 + v^2$.

Section – B

Answer any *three* questions **(3 × 8 = 24)**

4. Evaluate $\int \frac{dx}{4+5\cos x}$.
5. For what values of 'p' does the integral $\int_1^{\infty} \frac{1}{x^p} dx$ converge?
6. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$.
7. Evaluate the integral $\iint_R \sqrt{x^2+y^2} dA$, where $R = \{(x,y)/1 \leq x^2 + y^2 \leq 9, y \geq 0\}$ by changing to polar coordinates.

Section – C

Answer any *one* question **(1 × 20 = 20)**

8. (a) Prove that $\int_2^3 \sqrt{(x-2)(3-x)} dx = \frac{\pi}{8}$.

   (b) Using the Comparison test for improper integrals, determine whether the integral $\int_1^{\infty} \frac{dx}{x+e^{2x}}$ is convergent or divergent:

   (c) Find $\int_0^1 \left\{x \log \frac{1}{x}\right\}^{1/3} dx$. **(9+4+7)**

9. (a) Find the area of the surface of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

   (b) Evaluate the integral $\iiint_E (xz - y^3) dV$, where $E = \{(x,y,z)/-1 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 1\}$. **(10+10)**

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