STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/VL65

B.SC. DEGREE EXAMINATION, April 2021
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : **VECTOR SPACES AND LINEAR TRANSFORMATIONS**
TIME : 90 minutes MAXIMUM MARKS : 50

SECTION –A

Answer ***ALL*** the questions (3×2=6)

1. Let $V = \mathbb{R}^2$ be a vector space over the field $\mathbb{R}$ of real numbers. Check whether the subset $W_1 = \{(a, b) \mid a \geq 0, b \geq 0\} \subseteq V$ is a subspace of V.
2. Show that the orthogonal complement of a subspace of a vector space V is a subspace of V.
3. Construct an isomorphism from the vector space of symmetric 2×2 matrices onto $\mathbb{R}^3$.

SECTION –B

Answer ***ANY THREE*** questions (3×8=24)

4. Prove that if V is the internal direct sum of $U_1, U_2, \dots, U_n$, then V is isomorphic to the external direct sum of $U_1, U_2, \dots, U_n$.
5. Prove that if V is a vector space and $u, v \in V$, then $|\langle u, v\rangle| \leq \|u\| \, \|v\|$.
6. Prove that if V is finite-dimensional vector space over F, then $T \in A(V)$ is regular if and only if T maps V onto V.
7. Orthogonally diagonalize the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \end{bmatrix}$.

SECTION –C

Answer ***ANY ONE*** question (1×20=20)

8. (a) State and prove the homomorphism theorem for vector spaces.

   (b) Prove that if $v_1, v_2, \dots, v_n$ is a basis of the vector space V over F, and if $w_1, w_2, \dots, w_m$ in V are linearly independent over F, then $m \leq n$. (10+10)
9. (a) State and prove Gram-Schmidt orthogonalization process.

   (b) Let U be a vector space with bases B and B'. Let P be the transition matrix from B' to B. If T is a linear operator on U, having matrices A and A' with respect to the bases B and B' respectively, then obtain the relation between P, A and A'. (12 + 8)

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