STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16& thereafter)

SUBJECT CODE: 15MT/MC/CA65

B.Sc DEGREE EXAMINATION, APRIL - 2021.
END SEMESTER EXAMINATION BRANCH I - MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR – CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
TIME : 90 Mins MAX. MARKS : 50

Section – A
Answer ALL questions (3 × 2 = 6)

1. Verify the function $e^{x}(cosy - isiny)$ is analytic.
2. State and prove fundamental theorem of algebra.
3. Evaluate $\int_C \frac{dz}{z^2+4}$, where C is $|z-i| = 2$.

Section – B
Answer any THREE questions (3 × 8 = 24)

4. Show that $u = \log\sqrt{x^2+y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function $f(z)$.
5. Find the bilinear transformation which maps $z_1 = -i, z_2 = 0, z_3 = i$;

   $into\ \ w_1 = -1, w_2 = i, w_3 = 1.$
6. State and prove Cauchy's Residue theorem and find the residue of $\int_C \frac{3z-4}{z(z-1)}$, where C is $|z| = 2$.
7. State and prove Taylors series theorem for any analytic function.

Section – C
Answer any ONE question (1 × 20 = 20)

8. a) Brief on the elementary transformation of the function $w = e^{Z}$ in the complex plane.

   b) If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansions in the region

   (i) $0 < |z-1| < 4$, (ii) $|z-1| > 4$.

   c) Prove that any analytic function whose imaginary part is constant is itself a constant,

   (8+8+4)
9. a) State and prove Cauchy Integral formula and find $\int_C \frac{e^{az}}{z^{n+1}}$, where C is $|z| = \frac{1}{2}$.

   b) Prove that $I = \int_0^{\pi} \frac{ad\theta}{a^2+sin^2\theta} = \frac{\pi}{\sqrt{a^2+1}}$ $(a > 0)$.

   (10+10)

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   $into\ \ w_1 = -1, w_2 = i, w_3 = 1$