STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086

(For candidates admitted from the academic year 2019 – 20)

SUBJECT CODE: 19MT/PE/MS15

M. Sc. DEGREE EXAMINATION, APRIL 2021

BRANCH I - MATHEMATICS

FOURTH SEMESTER

COURSE : ELECTIVE **TIME : 90 MINUTES**

PAPER : MATHEMATICAL STATISTICS **MAX. MARKS : 50**

SECTION – A

ANSWER ***ALL*** THE QUESTIONS (2 × 2 = 4)

1.What is the distribution followed by the variable $X = \lambda Y + \mu, \lambda > 0$ if Y has the density function of the form $\frac{1}{2}e^{-|y|}$?

2. What are the conditions for an unbiased estimate U of the parameter Q to be most efficient?

SECTION – B

ANSWER ANY ***TWO*** QUESTIONS (2 × 6 = 12)

3. Define two-point distribution. Find the characteristic function of zero – one distribution and hence find the central moments μ_1, μ_2 of zero – one distribution.

4. Let $F_n(x), n = 1,2, \ldots$ be the distribution function of a random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 \; for \; x \leq 0 \\ 1 \; for \; x > 0 \end{cases}.$$

5. Define student's non-central t-statistic and derive its density function.

SECTION – C

ANSWER ANY ***TWO*** QUESTIONS (2 × 17 = 34)

6. The joint distribution of the dependent variables X, Y is given by the density (13)

$$f(x,y) = \begin{cases} \frac{1}{4}[1 + xy(x^2 - y^2)], |x| \leq 1, |y| \leq 1 \\ 0 \, , otherwise \end{cases}$$. Find the characteristic function of $Z = X + Y$.

b) Find mean and variance of Gamma distribution by using its characteristic function. (4)

7. a) Derive the distribution function of Chi-Square variate and State any two properties of Chi-square curve. (7)

b) State and prove Lindeberg – Levy theorem. (10)

8. State and prove the Rao-Cramer inequality.