STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 and thereafter)

SUBJECT CODE: 19MT/PC/MI24

M.Sc. DEGREE END SEMESTER EXAMINATION- MAY 2021

COURSE: CORE **TIME: 90 Minutes**
PAPER: MEASURE THEORY AND INTEGRATION **MAX.MARKS: 50**

SECTION – A
Answer all questions ($2 \times 2 = 4$)

1. Show that there exist uncountable sets of zero measure.
2. When does a class of sets said to be hereditary?

SECTION – B
Answer any two questions ($2 \times 6 = 12$)

3. Show that the class of all Lebesgue measurable sets, $\mathcal{M}$ is a σ- algebra.
4. Prove: Not every measurable set is a Borel set.
5. Show that if $\{A_i\}$ is a monotone sequence of sets, then $\lim A_i^y = (\lim A_i)^y$ and $\lim(A_i)_x = (\lim A_i)_x$.

SECTION –C
Answer any two questions ($2 \times 17 = 34$)

6.(a) Prove that the outer measure of an interval equals its length.
(b) Show that every non-empty open set has positive measure.
(c) Prove that there exists a positive set with respect to ν a signed measure on $[\![X, \mathcal{S}]\!]$ such that $A \subseteq E$ and $\nu(A) > 0$ where $E \in \mathcal{S}$ and $\nu(E) > 0$.

(8+3+6)

7.(a) If μ is a measure on a $\sigma -$ ring $\mathcal{S}$, then prove that the class $\bar{\mathcal{S}}$ of sets of the form $E \Delta N$ for any sets E, N such that $E \in \mathcal{S}$ while N is contained in some set in $\mathcal{S}$ of zero measure, is a $\sigma -$ ring and the set function $\bar{\mu}$ defined by $\bar{\mu}(E\Delta N) = \mu(E)$ is a complete measure on $\bar{\mathcal{S}}$.
(b) Show that if f is an integrable function, then $|\int f\, dx| \leq \int |f|\, dx$. When does equality occur?

(13+4)

8. (a) Prove that there exists uniquely defined measures ν^+ and ν^- on $[\![X, \mathcal{S}]\!]$ such that $\nu = \nu^+ - \nu^-$ and $\nu^+ \perp \nu^-$ where ν is a signed measure on $[\![X, \mathcal{S}]\!]$.
(b) State and prove Fubini's Theorem.

(9+8)