STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019–20 and thereafter)
SUBJECT CODE: 19MT/PC/LA24
M. Sc. DEGREE EXAMINATION - APRIL 2021
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : LINEAR ALGEBRA
TIME : 90 MINUTES **MAX. MARKS: 50**

SECTION – A
Answer ALL questions **(2×2=4)**

1. Prove that similarity is an equivalence relation on $A(V)$?
2. Define sesqui-linear form.

SECTION – B
Answer any TWO questions **(2×6=12)**

3. Prove that if the matrix $A \in F_n$ has all its characteristic roots in F, then there is a matrix $C \in F_n$ such that CAC^{-1} is a triangular matrix.
4. Let A be a 4×4 matrix with minimal polynomial $m(t) = (t^2 + 1)(t^2 - 3)$. Find the rational canonical form for A if A is a matrix over
   i) the rational field Q
   ii) the real field R
5. If $T : V \to W$ is a linear transformation, then prove that dim V = rank T + nullity T.

SECTION – C
Answer any TWO questions **(2×17=34)**

6. If $T \in A(V)$ is nilpotent, of index n_1, prove that a basis of V can be found such that the matrix of T in this basis has the form

$$\begin{bmatrix} Mn_1 & 0 & . & . & . & 0 \\ 0 & Mn_2 & . & . & . & 0 \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ 0 & 0 & . & . & . & Mn_r \end{bmatrix}.$$

   where $n_1 \geq n_2 \geq ... \geq n_r$ and where $n = n_1 + n_2 + ... + n_r = \dim_F V$.
7. (a) Prove that the elements S, $T \in A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

(b) Let V be a finite dimensional vector space over F and T be a linear operator on V. Also, if the minimal polynomial for T has the form $(x-c_1)(x-c_2)\ldots(x-c_k)$ where $c_1, c_2, \ldots, c_k$ are distinct elements of F, then prove that T is diagonalizable.

(10+7)

8. (a) Let V be a finite dimensional inner product space, T- a linear operator on V and B be an orthonormal basis for V. Suppose that the matrix A of T in the basis B is upper triangular then prove that T is normal iff A is a diagonal matrix.

(b) Let f be a form on a finite dimensional vector space V and let A be the matrix of f in an ordered basis B. Prove that f is a positive form if and only if $A = A^*$ and the principal minors of A are all positive. **(8+9)**

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