STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PC/TO34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Let X and Y be metric spaces and f a mapping of X into Y show that f is continuous at x_0 if and only if $x_n \to x_o \Rightarrow f(x_n) \to f(x_0)$.
2. Define separable metric space and a second countable space. Show that every separable metric space is second countable.
3. Show that every closed and bounded subspace of the real line is compact.
4. Define Bolzano-Weierstrass property. Show that a metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property.
5. Define totally bounded set. Show that every sequentially compact metric space is totally bounded.
6. Show that every compact Hausdorff space is normal.
7. Show that the product of any non-empty class of connected spaces is connected.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. (i) State and prove Cantor's intersection theorem.
   (ii) Let X and Y be metric spaces and f a mapping of X into Y show that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

9. (i) Show that the product of any non-empty class of compact spaces is compact.
   (ii) State and prove Lebesgue's covering lemma.

10. (i) State and prove Urysohn's lemma.
    (ii) Show that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.

11. State and prove Urysohn imbedding theorem.

12. (i) Show that a subspace of the real line R is connected if and only if it is an interval.
    (ii) Let X be a compact Hausdorff space , show that X is totally disconnected if and only if it has an open base whose sets are also closed.