STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : REAL ANALYSIS – PART - I
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Prove that every non empty open set S in $\Re^1$ is the union of a countable collection of disjoint component intervals of S.
2. Define the following
   (i) accumulation point
   (ii) adherent point
   (iii) metric space
3. Prove that in Euclidean space $\Re^k$ every Cauchy sequence is convergent.
4. Let $f: S \to T$ be a function from one metric space (S, d_s) to another (T, d_T). Let A be a compact subset of S and assume that f is continuous on A. Then prove that f is uniformly continuous on A.
5. a) Define point wise convergence.
   b) Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, the prove that the limit function f is also continuous at c.
6. a) Show that a function can have a finite directional derivative $\bar{f}'(\bar{c};\bar{u})$ for every $\bar{u}$ but may fail to be continuous at $\bar{c}$ with a suitable example.
   b) Prove that if $\bar{f}$ is differentiable at $\bar{c}$ then $\bar{f}$ is continuous at $\bar{c}$.
7. a) Define the Jacobian matrix of $\bar{f}$ at $\bar{c}$.
   b) For each of the following functions, verify that the mixed partial derivatives $D_{1,2}f$ and $D_{2,1}f$ are equal.
   (i) $f(x, y) = \log(x^2 + y^2),\ (x, y) \neq (0,0)$
   (ii) $f(x, y) = \tan\left(x^2 / y\right),\ y \neq 0$

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) State and prove Bolzano Weierstrass theorem.

   b) Let (S,d) be a metric subspace of (M,d) and let X be a subset of S, then prove that X is open if and only if $X = A \cap S$ for some set A which is open in M.

9. a) Let $f : S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). Prove that f is continuous on S if and only if for every open set Y in T the inverse image $f^{-1}(Y)$ is open in S.

   b) Prove that every arc wise connected set in $\Re^n$ is connected and the converse is true for open connected sets.

10. a) Prove that a contraction of a complete metric space S has a unique fixed point p.

    b) State and prove Cauchy condition for uniform convergence.

11. a) State and prove the chain rule for computing the total derivative of he composition of two functions $\overline{f}$ and $\overline{g}$.

    b) State and prove the Mean value theorem for vector valued differentiable functions.

12. a) If both partial derivatives $D_r \overline{f}$ and $D_k \overline{f}$ exist in an n - ball $B(\overline{c};\delta)$ and if both are differentiable at $\overline{c}$ then prove that $D_{r.k}\overline{f}(\overline{c}) = D_{k,r}\overline{f}(\overline{c})$.

    b) State and prove Taylor's formula for functions from $\Re^n$ to $\Re$.

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