STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PC/ME14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Derive the conservation theorem of total energy for a system of particles.
2. Using calculus of variations obtain the curve joining two fixed points for which the surface of revolution obtained by revolving the curve about the Y-axis is a minimum.
3. State and prove Euler's theorem on the motion of a rigid body with one point fixed.
4. Define Routh's function for a system and explain Routhian procedure in solving a mechanical problem.
5. Define Corioles force and hence obtain the amount of deflection from the vertical of a freely falling particle due to this force.
6. Obtain the necessary and sufficient condition in simplectic notation for a set of transformations to be canonical.
7. State and prove Jacobi's identify relating to Poisson brackets.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Obtain Lagrange's equations for nonholonomic systems.
   b) Illustrate the Lagrange multiplier method by considering the rolling, without slipping, of a hoop down an inclined plane.

9. a) Define inertia tensor. Obtain eigen values of inertia tensor and principal axes transformation.
   b) Write the problem of central force motion of two mass points in Hamiltonian formulation, eliminating the cyclic variables, and reducing the problem of quadratures.

10. State and prove the Principle of Least Action explaining clearly the symbols used and the assumptions made. Further deduce Jacobi's form of this principle.

11. a) Define canonical transformation equations and the generating function. Show how canonical transformation equations can be obtained when the generating function takes any one of the four standard forms.

b) Show that the transformation $Q = \sqrt{q}\cos 2p$, $P = \sqrt{q}\sin 2p$ is a canonical transformation and that the function which generates this transformation is $F_3 = \dfrac{-Q^2 \tan 2p}{2}$.

12. a) Define Poisson and Lagrange brackets and show that they are canonical invariants.

b) By any method, show that the following transformation is canonical.

$$x = \frac{1}{\alpha}\left(\sqrt{2P_1}\sin Q_1 + P_2\right), \quad P_x = \frac{\alpha}{2}\left(\sqrt{2P_1}\sin Q_1 - Q_2\right),$$

$$y = \frac{1}{\alpha}\left(\sqrt{2P_1}\cos Q_1 + Q_2\right), \quad P_y = -\frac{\alpha}{2}\left(\sqrt{2P_1}\sin Q_1 - P_2\right)$$

Where α is some fixed parameter.