STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PC/GT14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**GRAPH THEORY**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. a) Define: Graph Isomorphism. Give an example

   b) Prove that in a graph the number of vertices of odd degree is even.

2. a) Prove that a graph is bipartite if and only if it contains no odd cycles.

   b) Show that if $\delta(G) \geq 2$, then G contains a cycle.

3. a) Define a spanning tree of a graph G and illustrate with an example.

   b) A vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$. Prove.

4. a) Define the connectivity κ and edge connectivity κ' of a graph G. Draw a graph for which $\kappa < \kappa'$.

   b) Show that if G is k - edge connected then $\varepsilon \geq \frac{k\gamma}{2}$.

5. a) Give an example of a graph that is Eulerian but not Hamiltonian

   b) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \frac{\gamma}{2}$, then G is Hamiltonian. Prove.

6. Prove that $\alpha + \beta = \gamma$ with usual notations.

7. Define a planar graph and prove that K_5 is non-planar.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) A graph G with γ vertices and ε edges has t vertices of degree m and all other vertices of degree n. Show that $(m-n)t+\gamma_n = 2\varepsilon$.

   b) If G is a tree, prove that any two vertices are connected by a unique path.

   c) If G be a graph with γ vertices and $\gamma - 1$ edges. Show that the following are equivalent.

   (i) G is connected (ii) G is acyclic (iii) G is a tree

9. a) With usual notations prove that $K \leq K' \leq \delta$.

   b) State and prove a necessary and sufficient condition for a graph to be Eulerian.

10. a) for any graph G with 6 vertices prove that G or $\overline{G}$ contains a triangle.

    b) show that for all k and l, $r(k,l) = r(l,k)$.

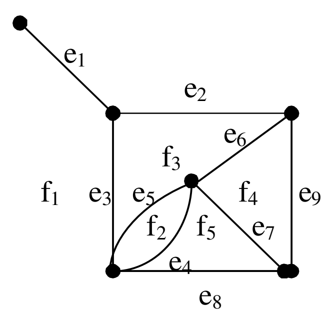
    c) State and prove Ramsey's theorem.

11. a) State and prove Brooke's theorem.

    b) Prove that every critical graph is a block.

12. a) Prove that every planar graph is 5-colourable.

    b) Define dual of a graph. Draw the dual of the following graph.



⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄