STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE: MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : COMPLEX ANALYSIS – PART II
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Prove that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear function of $\log r$.
2. Prove that $\xi(s) = \frac{1}{2}s(1-s)\pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s)$ is entire and deduce the Legendre's duplication formula.
3. Show that a family F is normal if and only if its closure $\overline{F}$ with respect to the distance function is compact.
4. Prove that a continuous function u(z) which satisfies mean-value property is necessarily harmonic.
5. Show that the sum of residues of an elliptic function is zero.
6. Prove that the weierstrass p-function can be represented in the form

$$p(z) = \frac{1}{z^2} + \sum_{w \neq 0}\left(\frac{1}{(z-w)^2} - \frac{1}{w^2}\right)$$

7. Prove that $p(z) - p(u) = \frac{\sigma(z-u)\sigma(z+u)}{\sigma(z)^2\sigma(u)^2}$.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Show that (i) $P_U(z)$ is harmonic for $|z|<1$.

(ii) $\lim_{z \to e^{i\theta_o}} P_U(z) = U(\theta_0)$ if U is continuous at θ_0.

b) For $\sigma = \operatorname{Re} s > 1$, prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}\left(1 - p_n^{-s}\right)$ where $p_1, p_2, \ldots p_n$ is an ascending sequence of primes.

9. State and prove Arzela's theorem.

10. State and prove Riemann mapping theorem.

11. a) Describe with proof all discrete module.

b) Show that the zeros $a_1, a_2, \ldots a_n$ and poles $b_1, b_2, \ldots b_n$ of an elliptic function satisfy $a_1 + a_2 + \ldots + a_n \equiv b_1 + b_2 + \ldots + b_n \pmod M$

12. a) Derive the differential equation satisfied by weierstrass p-function.

b) Prove that $p(z+u) = -p(z) - p(u) + \frac{1}{4}\left(\frac{p'(z) - p'(u)}{p(z) - p(u)}\right)^2$.

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