STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PC/AL14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA – PART - I
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. If G is a finite group and if $a \in G$, prove that the number of elements conjugate to a in G is equal to the index of the normalizer of a in G.
2. Show that no group of order 30 is simple.
3. If p is a prime number of the form $4n+1$, prove that $p = a^2 + b^2$, for some integers a, b.
4. State and prove the Eisenstein Criterion about the irreducibility of a polynomial with integer coefficients.
5. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F.
6. If R is an Euclidean ring, prove that any two elements $a, b \in R$ have a greatest common divisor of the form $\lambda a + \mu b$, for some $\lambda, \mu \in R$.
7. a) Define a nilpotent linear transformation.
   b) show that a nilpotent linear transformation does not admit any non-zero eigen value.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. Prove that every finite abelian group is the direct product of cyclic groups.
9. a) Prove that *J(i)*, the ring of Gaussian integers, is a Euclidean ring.
   b) Show that any ideal in a Euclidean ring is a principal ideal.
10. a) If p is prime, prove that the polynomial $1 + x + x^2 + ... + x^{p-1}$ is irreducible over the field of rational numbers.
    b) If R is unique factorization domain and if p(x) is a primitive polynomial in R[X], prove that p(x) can be factored in a unique way as the product of irreducible elements in R[x].
11. Prove that two nilpotent linear transformation are similar if and only if they have the same invariants.
12. If $T \in A(V)$ has all its characteristic roots in F, then prove that there exists a basis of V in which the matrix of T is triangular.

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