STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/MC/AS54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2009
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10 X 2 = 20)**
ANSWER ALL THE QUESTIONS

1. Define an abelian group. Give an example.
2. Prove that (i) The identity element of a group G is unique
   (ii)inverse of an element 'a' in a group is unique
3. State the cancellation Laws in a group G.
4. Define a cyclic group. Give an example
5. Prove that intersection of two subgroups is again a subgroup.
6. Let $\phi : G \rightarrow \overline{G}$ be a group homomorphism. Prove that (i) $\phi(e) = \overline{e}$ and
   (ii) $\phi\left(a^{-1}\right) = \left(\phi(a)\right)^{-1}$ where e and $\overline{e}$ are identities of G and $\overline{G}$ and $a \in G$.
7 Define a commutative ring.
8 Define an ideal of a ring.
9 Define a field.
10 Define maximal ideal.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Prove that there is a one-to-one correspondence between any two right cosets of H in G.
12. State and prove Lagrange's Theorem.
13. Prove that a subgroup N of G is normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
14. State and prove Cayley's Theorem.
15. Prove that a finite integral domain is a field.
16. Prove that intersection of two ideals of a ring is again an ideal.
17. If U and V are ideals of R let $U + V = \{u + v / u \in U, v \in V\}$. Prove that $U + V$ is an ideal.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) If H and K are subgroups of a group G of orders $o(H)$ and $o(K)$ respectively.

    Prove that $o(HK) = \dfrac{o(H)\,o(K)}{o(H \cap K)}$.

    (b) If G is finite group and $a \in G$. Then prove that $o(a) / o(G)$.

19. (a) $\phi : G \to G$ be a group homomorphism of G onto $\overline{G}$ with Kernel K . Then prove that (i) K is a normal subgroup of G.

    (ii) Prove that G/K is isomorphic to G.

    (b) Prove that any field is an integral domain.

20. Prove that every integral domain can be imbedded in a field.