STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE : 19MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 × 2 = 10)**
ANSWER ALL QUESTIONS

1. If A is an open and B is closed then prove that $A \setminus B$ is open and $B \setminus A$ is closed.
2. Give an example of function of a bounded variation on $[0,1]$.
3. Define a step function on $[a,b]$.
4. State Taylor's formula for functions from $\mathbb{R}^n$ to $\mathbb{R}$.
5. State implicit function theorem.

SECTION – B **(5 × 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Prove that a set S is closed if and only if it contains all its adherent points.
7. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows: $V(x) = V_f(a,x)$ if $a < x \le b$, $V(a) = 0$. Prove that
   (a) V is an increasing function on $[a,b]$.
   (b) $V - f$ is an increasing function on $[a,b]$.
8. If f is continuous on $[a,b]$ and if α is of bounded variation on $[a,b]$, then prove that $f \in R(\alpha)$on $[a,b]$
9. If f is differentiable at c with total derivative existing, then prove that f is continuous at c.
10. Let A be an open subset of $\mathbb{R}^n$ and assume that $\bar{f}: A \to \mathbb{R}^n$ has continuous partial derivatives $D_j f_i$ on A. If $J_f(\bar{x}) \neq 0$ for all $\bar{x}$ in A then prove that $\bar{f}$ is an open mapping.
11. If $f \in R(\alpha)$ on $[a,b]$, then prove that $\alpha \in R(f)$ on $[a,b]$ and $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.
12. For some integer $n \ge 1$, let f have a continuous nth derivative in the open interval (a,b). Suppose also that for some interior point c in (a,b), we have $f'(c) = f''(c) = ... = f^{(n-1)}(c) = 0$, but $f^n(c) \neq 0$. Prove that for n even, f has a local minimum at c if $f^n(c) > 0$, and a local maximum at c if $f^n(c) < 0$.

SECTION – C **(3 × 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. (a) State and prove Cantor intersection theorem. (8 Marks)
    (b) Let F be an open covering of a closed and bounded set A in R^n. Prove that a finite subcollection of F also covers A. (12 Marks)

14. Let f be of bounded variation on $[a, b]$. If $x \in (a, b]$, let let $V(x) = V_f(a, x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Prove also that the converse is also true.

15. (a) Assume $f \in R(\alpha)$ on $[a,b]$ and assume that α has a continuous derivative α' on $[a,b]$. Prove that the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and

    $$\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx.$$ (15 Marks)

    (b) State and prove first mean value theorem for Riemann – Stieltjes integrals. (5 Marks)

16. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Prove that the composite function $h = f \circ g$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$, the composition of the linear function $f'(b)$ and $g'(a)$.

17. State and prove the inverse function theorem.