STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE: 19MT/PC/OD14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ORDINARY DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**
ANSWER ALL THE QUESTIONS

1. Find the general solution of $x^{(4)} - 16x = 0$.
2. Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.
3. Define a regular singular point.
4. Compute the first two consecutive approximations for the solution of the equation $x' = x^2$, $x(0) = 1$.
5. Define a Green function.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Solve: $x^m + x^n = 0$, $x(0) = 1$, $x'(0) = 0$ and $x''(0) = 1$.
7. Solve: Consider the linear system $x' = A(t)x$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$.

   Show that the matrix $\Phi(t) = \begin{bmatrix} e^{-3t} & te^{-3t} & \frac{e^{-3t}t^2}{2!} \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix.
8. If P_n is a Legendre polynomial then prove that $\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}$.
9. Let $x(t) = x(t, t_0, x_0)$ and $x^*(t) = x(t, t_0^*, x_0^*)$ be a solution of the IVP $x' = f(t,x)$, $x(t_0) = x_0$ and $x' = f(t,x)$, $x(t_0^*) = x_0^*$ respectively on an interval $a \le t \le b$. Let $(t, x(t))$, $(t, x^*(t))$ lie in a domain D for $a \le t \le b$. Further, let $f \in Lip(D, K)$ be bounded by L in D. Prove that for any $\in > 0$, there exists a $\delta = \delta(\in) > 0$ such that $|x(t) - x^*(t)| < \in$, $a \le t \le b$ whenever $|t - t_0^*| < \delta$ and $|x - x_0^*| < \delta$.

10. Find the eigen values and eigen functions of the BVP $x''+\lambda x=0$, $x(0)=0$, $x'(1)=0$.

11. Assume that

(i) A, B are finite real numbers;

(ii) The functions $p'(t)$, $q(t)$ and $r(t)$ are real valued continuous functions on $[A,B]$.

For the parameters λ, μ $(\lambda \neq \mu)$, let x and y be the corresponding solutions of $(px')'+qx+\lambda rx=0$, $A \le t \le B$ such that $\left[pW(x,y)\right]_A^B=0$, where $W(x,y)$ is the Wronskian of x and y. Prove that $\int_A^B r(s)x(s)y(s)=0$.

12. Prove that $J_n(t)=\frac{1}{\pi}\int_0^{\pi}\cos(n\theta - tsin\theta)\,d\theta$.

SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. (a) Let $b_1, b_2, ..., b_n$ be real or complex valued functions defined and continuous on an interval I and $\phi_1, \phi_2, ..., \phi_n$ are n solutions of the equation $x^{(n)}(t)+b_1(t)x^{(n-1)}(t)+....+b_n(t)x(t)=0$, $t \in I$ existing on I. Prove that these n solutions are linearly independent on I if and only if the Wronskian $w(t) \neq 0$ for every $t \in I$. (15 Marks)

(b) Prove that there are three linearly independent solutions of the third order equation $x'''+b_1(t)x''+b_2(t)x'+b_3(t)x=0$, $t \in I$, where b_1, b_2 and b_3 are functions defined and continuous on an interval I. (5 Marks)

14. Let $A(t)$ be an $n \times n$ matrix that is continuous in t on a closed and bounded interval I. Prove that there exists a solution to the IVP $x'=A(t)x$, $x(t_0)=x_0$; $t, t_0 \in I$ and also prove that this solution is unique.

15. (a) Find the power series solutions of the equation $x''(t)+K\sin x(t)=0$ with initial conditions $x(0)=\frac{\pi}{6}$ and $x'(0)=0$. (5 Marks)

(b) Solve the Legendre equation $(1-t^2)x''-2tx'+p(p+1)x=0$, where p is a real number by power series method. (15 Marks)

16. State and prove Picard's theorem.

17. Use the method of separation of variables to solve the boundary value problem

$$\frac{\partial u}{\partial t}(x,t)=k\frac{\partial^2 u}{\partial x^2}(x,t),\ (0<x<c,\ t>0)$$

$$\frac{\partial u}{\partial t}(0,t)=0,\ \frac{\partial u}{\partial x}(c,t)=0,\ (t>0) \text{ and } u(x,0)=f(x),\ (0<x<c).$$

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