STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE: 19MT/PC/GT14

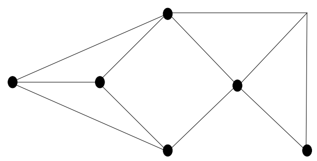
M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**GRAPH THEORY**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A **(5 X 2 = 10)**
ANSWER ALL THE QUESTIONS

1. Give the incidence and adjacency matrix for a complete graph on 5 vertices.
2. Find the connectivity and edge-connectivity for the following graph:



3. When do you say a graph G is critical?
4. State Kuratowski's theorem.
5. Draw a hypercube network of dimension 4.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

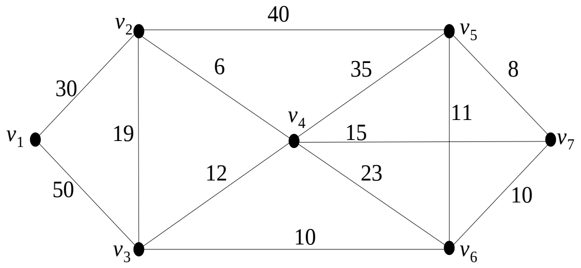
6. Give a characterization for bipartite graphs using the concept of a cycle.
7. Prove that a vertex v of tree G is a cut-vertex of G if and only if $d(v) > 1$.
8. Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.
9. If G is a connected graph which is not an odd cycle, then show that G has a 2-edge coloring in which both colors are represented at each vertex of degree at least two.
10. Prove the theorem that gives the necessary and sufficient condition for a graph G to have a set consisting of any two adjacent vertices as a minimal dominating set of G.
11. Explain the basic principles of network designing.
12. Let G be a graph with order n. Prove that for any $\theta \in Aut(G)$, its restriction to X is an isomorphism between $G[X]$ and $G[\theta(X)]$ for any non-empty $X \subseteq V(G)$, where

$$\theta(X) = \{y \in V(G) : y = \theta(x), x \in X\}.$$

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. State Dijkstra's algorithm. Use it to find the shortest path between v_1 and all other vertices in the following graph.



14. (i) Show that a bipartite graph G with bipartition (X, Y) has a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.

    (ii) If $\alpha'(G)$ and $\beta'(G)$ denotes the edge independence number and edge covering number of a graph G respectively, then prove that $\alpha'(G) + \beta'(G) = \nu$ for $\delta > 0$. (10+10)

15. (i) State and prove Brook's theorem.

    (ii) For a simple graph G, prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$. (10+10)

16. (i) Derive Euler's formula.

    (ii) Let G be a nonplanar connected graph that contains no subdivision of $K_{3,3}$ or K_5 having a few edges. Then prove that G is simple and 3-connected.

17. Define a De Brujin, Kautz, Circulant networks and state basic properties of these networks.