STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE : 19MT/PC/AA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ABSTRACT ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**
ANSWER ALL QUESTIONS

1. List the conjugate classes of the symmetric group S_3.
2. Define a unit in a ring and write down all the units of the ring Z of all integers.
3. Define a unique factorization domain and give an example.
4. Does there exist an extension field K of the field R of real numbers which is contained in the field C of Complex numbers? Give reasons for your answer.
5. Define a normal extension of a field and give an example.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Prove that any group of order p^2, p a prime, is abelian.
7. Prove that any prime number of the form $4n + 1$ can be expressed as a sum of two squares.
8. State and prove the Eisenstein Criterion about the irreducibility of a polynomial with integer coefficients.
9. If a and b in K are algebraic over F of degrees m and n, respectively, and if m and n are relatively prime, prove that $F(a, b)$ is of degree mn over F.
10. Prove that a group G is solvable if and only if $G^{(k)} = (e)$ for some k.
11. If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and satisfies $o\big(G(K, F)\big) \leq [K{:}F]$.
12. Define a simple extension of a field. Is a simple extension a finite extension? Justify your answer.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. (a) Prove that in a finite group G , the number of elements conjugate to an element a in G is equal to the index of the normalizer of a in G.

    (b) Prove that any two p-Sylow subgroups of a finite group G are conjugates.

    (10+10)

14. (a) Prove that the ring of Gaussian integers is a Euclidean ring.

    (b) Prove that an ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R. (10+10)

15. If R is a unique factorization domain, prove that the polynomial ring $R[x]$ is also a unique factorization domain, by proving all the necessary results that are used in the proof.

16. (a) Prove that an element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

    (b) stateand Prove the Remainder theorem. (15 + 5)

17. Prove that K is a normal extension of F if and only if K is a splitting field of a polynomial over F.