STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/TO34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 ×2 = 10)

1. Show that finite union of closed sets is closed.
2. Define a locally path connected space.
3. Give an example of a topological space which is not compact.
4. State Urysohn'smetrization theorem.
5. State the Tychonoff theorem.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 × 6 = 30)

6. Let X be a subspace of X and $A \subset Y$. Show that A is closed in Y if and only if it equals the intersection of closed set of X with Y.
7. A subspace X is locally connected if and only if for every open set U of X, each component of U is open in X.
8. State and prove the Lebesgue number lemma.
9. Prove that every metrizable space is normal.
10. Let X and Y be topological spaces. Let $f: X \to Y$. Then show that the following are equivalent.
    a. f is continuous
    b. For every subset A of X, one has $f(\bar{A}) \subset \overline{F(A)}$.
    c. For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.
11. State and prove the intermediate value theorem.
12. Show that a subspace of a Hausdorff space is Hausdorff and a product of Hausdorff space is Hausdorff.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 ×20 = 60)**

13. (a) Let (X, τ) be a topological space and $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each $x \in U$, there exists $C \in \mathcal{C}$ such that $x \in C \subset U$. Show that $\mathcal{C}$ is a basis for the topology τ on X.
    (b) If A is a subset of the topological space X, then prove that $x \in \bar{A}$ if and only if every open set U containing x intersect A. Deduce that $\bar{A} = A \cup A'$, where A' is the set of all limit points of A.

14. (a) If L is a linear continuum in the order topology, then show that L is connected.
    (b) Prove that a finite Cartesian product of connected spaces is connected.
    (c) Show that $\mathbb{R}^{\omega}$ is not connected in the box topology.

15. (a) Show that every closed subset of a compact space is compact.
    (b) Let X be a simply ordered set having the least upper bound property. In the order topology, show that each closed interval in X is compact.

16. State and prove Tietz extension theorem.

17. (a) Let $f: A \rightarrow \prod_{\alpha \in J} X_\alpha$ defined by $f(a) = \left(f_\alpha(a)\right)_{\alpha \in J}$ where $f_\alpha: A \rightarrow X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Show that f is continuous if and only if each function f_α is continuous.
    (b) State and prove the pasting lemma.

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