STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE: 15MT/PC/GT34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A (5 X 2 = 10)
ANSWER ALL THE QUESTIONS

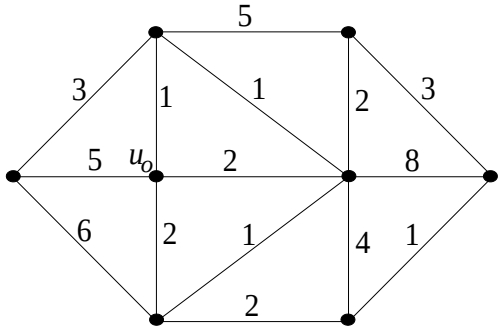
1. When are two graphs said to be identical?
2. Define an M-alternating path in a graph G.
3. Define chromatic number of a graph G.
4. Define a directed graph.
5. Explain graph embedding problem.

SECTION – B (5 X 6 = 30)
ANSWER ANY FIVE QUESTIONS

6. Prove that an edge e of a graph G is a cut-edge of G if and only if e is contained in no cycle of G.
7. With usual notations prove that $\kappa \leq \kappa' \leq \delta$.
8. If G is a k-regular bipartite graph with $k > 0$, then prove that G has a perfect matching.
9. Prove that no vertex cut is a clique in critical graph.
10. Show that $\nu - \varepsilon + \phi = 2$ for a connected plane graph.
11. (i) State Kuratowski's theorem
    (ii) Prove that at least one of H_1 and H_2 is nonplanar for a nonplanar graph G. (2+4)
12. Define a hypercube network Q_n and state some fundamental properties of Q_n.

SECTION – C (3 X 20 = 60)
ANSWER ANY THREE QUESTIONS

13. State Dijkstra's algorithm and use it to find the shortest distance between u_0 and all other vertices in the following graph.



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14. (i) Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path. (15 marks)

(ii) Show that a set $S \subseteq V$ is an independent set of a graph G if and only if $V - S$ is a covering of G. (5 marks)

15. (i) Show that if G is a simple graph, then $\pi_k(G) = \pi_k(G - e) - \pi_k(G \cdot e)$ for any edge e of G. (10 marks)

(ii) For a simple graph G, prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$. (10 marks)

16. (i) Prove that every planar graph is 5-vertex colorable. (10 marks)

(ii) Prove that a digraph D contains a directed path of length $\chi - 1$. (10 marks)

17. Explain the basic principles in the process of design of an interconnection networks.