STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE: 15MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION–A

ANSWER ALL QUESTIONS: **(5×2=10)**

1. What is the necessary and sufficient condition for a line integral to depend only on the end points?
2. Why is the integral of an exact differential over any cycle zero?
3. Find $\Gamma\left(\frac{1}{2}\right)$.
4. Define equi-continuity.
5. Give any two applications of conformal mapping in Fluid Dynamics.

SECTION–B

ANSWER ANY FIVE QUESTIONS: **(5×6=30)**

6. Define index of a point a with respect to a curve γ and prove that a function of a index $n(\gamma, a)$ is constant in each of the regions determined by γ and zero in the unbounded region.
7. Derive Poissons' formula.
8. Prove that the infinite product $\prod_1^\infty(1 + a_n)$ with $1 + a_n \neq 0$ converges simultaneously with the series $\sum_1^\infty \log(1 + a_n)$ whose terms represent the values of the principal branch of the logarithm.
9. How can the Riemann Zeta function be extended to the whole plane?
10. Show that the family $\mathfrak{I}$ is normal $\Leftrightarrow$ its closure with respect to a distance function is compact.
11. Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ tends to the boundary of Ω then prove that $\{f(z_n)\}$ tends to the boundary ofΩ'.
12. What is known as the Schwarz triangle function?

SECTION–C

ANSWER ANY THREE QUESTIONS: (3×20 =60)

13. a) State and prove Cauchy's theorem for a rectangle.

    b) If the piecewise differentiable closed curve γ does not pass through a point a then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$. (12+8)

14. a) If $f(z)$ is analytic in Ω, then prove that $\int_{\gamma} f(z)dz = 0$ for every cycle γ which is homologous to zero in Ω.

    b) State and prove the Reflection Principle. (8+12)

15. Find the representation for Euler's Gamma function.

16. State and prove Arzela's theorem.

17. State and prove the Riemann mapping theorem.