

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA AND TRIGONOMETRY
TIME : 3 HOURS MAX. MARKS : 100

SECTION – A (10X2=20)
ANSWER ANY TEN QUESTIONS

1. Solve $x^3 - 3x^2 - 4x + 12 = 0$ given that the sum of two roots is '0'.
2. If α, β, γ are the roots of $x^3 - 6x^2 + 11x - 21 = 0$, find the value of $\Sigma \alpha^2$.
3. Prove that $\frac{e+1}{e-1} = \frac{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots}{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}$.
4. Find the coefficient of x^n in $(3 + 2x)e^{3x}$.
5. State Cayley Hamilton theorem.
6. Find the characteristic equation of $a = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$.
7. What is the expansion of $\cos n\theta$.
8. Expand $\tan \theta$ in terms of θ .
9. Prove that $\cosh^2 x - \sinh^2 x = 1$.
10. Find the value of $\log(1 - i\sqrt{2})$.
11. Find the eigen value of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.
12. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} \dots$ show that $x = y + \frac{y^2}{2!} + \frac{y^4}{4!} + \dots$

SECTION – B (5X8=40)
ANSWER ANY FIVE QUESTIONS

13. Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and hence solve the equation.
14. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if two of the roots are in the ratio 3 : 2.
15. Prove that $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots \dots \infty = \frac{1}{e}$.
16. Prove that $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \dots \infty = \log 2 - \frac{1}{2}$.
17. Verify Cayley – Hamilton theorem for $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and hence find its inverse.
18. Express $\frac{\cos 7\theta}{\cos \theta}$ in power of $\cos \theta$.
19. If $\tan(\alpha + i\beta) = x + iy$, prove that $x^2 + y^2 + 2x \cot 2\alpha = 1$.

SECTION – C
ANSWER ANY TWO QUESTIONS

(2X20=40)

20. a) If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ form the equation whose roots are $\beta + \gamma - 2\alpha, \gamma + \alpha - 2\beta, \alpha + \beta - 2\gamma$.

b) Sum to infinity the series $\frac{1}{6} + \frac{5}{6.9} + \frac{5.7}{6.9.12} + \frac{5.7.9}{6.9.12.15} + \dots$

21. a) Diagonalise the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (14+6)

b) Prove that $\lim_{x \rightarrow 0} \frac{\log(\cos x) + \frac{1}{4} - \frac{1}{4} \cos 2x}{x^4} = -\frac{1}{4}$.

22. a) Show that $\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$.

b) Expand $\sin^3 \theta \cos^4 \theta$ in terms of series of multiples of θ .

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