STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16 and thereafter)

SUBJECT CODE: 15MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A
Answer All the questions (10 x 2 = 20)

1. If $\vec{A} = 5u^2\vec{\imath} + u\vec{\jmath} - u^3\vec{k}$ and $\vec{B} = sinu\vec{\imath} - cosu\vec{\jmath}$, find $\frac{d}{du}(\vec{A}\cdot\vec{B})$.
2. Suppose $\phi(x,y,z) = 3x^2y - y^3z^2$. Find $\nabla\phi$ at the point $(1,-2,-1)$.
3. Determine the constant a so that the following vector is solenoidal. $\vec{V} = (-4x - 6y + 3z)\vec{\imath} + (-2x + y - 5z)\vec{\jmath} + (5x + 6y + az)\vec{k}$.
4. Find a unit normal to the surface $2xy^2z - x^2yz^2 = 1$ at the point (1,1,1).
5. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = -3x^2\vec{\imath} + 5xy\vec{\jmath}$ and C is the curve in the xy-plane, $y = 2x^2$, from (0,0) to (1,2).
6. Define a conservative field.
7. Write down the Frenet-Serret formulae.
8. Define osculating plane.
9. State Stokes' theorem.
10. Evaluate $\iint_S \vec{r}\cdot\hat{n}\, dS$ where S is a closed surface.

SECTION-B
Answer any FIVE questions (5 x 8 = 40)

11. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = -t - 5$ where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\vec{\imath} - 2\vec{\jmath} + 2\vec{k}$.
12. Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1,-2,1)$.
13. Find the total work done in moving a particle in the force field given by $\vec{F} = z\vec{\imath} + z\vec{\jmath} + x\vec{k}$ along the helix C is given by $x = cost$, $y = sint$, $z = t$ from $t = 0$ to $t = \frac{\pi}{2}$.
14. Prove that a cylindrical coordinate system is orthogonal.
15. Use Gauss divergence theorem to evaluate $\iint_S \vec{F}\cdot\hat{n}\, dS$ where $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
16. Suppose $\vec{A}$ and $\vec{B}$ are differential functions of a scalar u. Prove that (i) $\frac{d}{du}(\vec{A}\cdot\vec{B}) = \vec{A}\cdot\frac{d\vec{B}}{du} + \frac{d\vec{A}}{du}\cdot\vec{B}$, (ii) $\frac{d}{du}(\vec{A}\times\vec{B}) = \vec{A}\times\frac{d\vec{B}}{du} + \frac{d\vec{A}}{du}\times\vec{B}$.
17. Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

SECTION-C

Answer any TWO questions **(2 x 20 = 40)**

18. (a) Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.Show that (i) $\nabla \ln|\vec{r}| = \frac{\vec{r}}{r^2}$, (ii) $\nabla r^n = nr^{n-2}\vec{r}$. (10)

(b) Show that (i) For a scalar ϕ , $\nabla \times (\nabla \phi) = 0$

(ii) For a vector $\vec{A}$, $\nabla \cdot (\nabla \times \vec{A}) = 0$. (10)

19. (a) Let $\vec{V} = (-4x - 3y + az)\vec{i} + (bx + 3y + 5z)\vec{j} + (4x + cy + 3z)\vec{k}$. Find the constants a, b and c so that the vector $\vec{V}$ is irrotational and hence express $\vec{V}$ as the gradient of a scalar function. (10)

(b) Evaluate $\iint_S \vec{A} \cdot \hat{n}\, dS$ where $\vec{A} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is the part of the plane $2x + 3y + 6z = 12$, which is located in the first octant. (10)

20. (a) Evaluate $\iiint_V (x^2 + y^2 + z^2)\, dx\, dy\, dz$ where V is a sphere having center at the origin and radius equal to a. (10)

(b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (10)