STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/RA55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**	
PAPER	**:**	**PRINCIPLES OF REAL ANALYSIS**	
TIME	**:**	**3 HOURS**	**MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ALL THE QUESTIONS

1. When do you say that a function is continuous at a point on the real line ?
2. Define Monotonic functions.
3. Define : Metric Space.
4. Define : Open set.
5. When is a subset of a metric space said to be bounded?
6. What is a Complete metric space ?
7. When is a metric space said to have the Heine Borel property ?
8. Define : Compact metric space.
9. Define upper sum and lower sum for a bounded function *f*.
10. State Rolle's Theorem.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

11. Show that if $lim_{x \to a} f(x) = L$ and $lim_{x \to a} g(x) = M$, then $lim_{x \to a}[\, f(x) + g(x)\,] = L + M.$
12. If G_1 and G_2 are open subsets of the metric space M , prove that $G_1 \cap G_2$ is also open.
13. Prove that if the subset A of the metric space M is totally bounded , then A is bounded.
14. If (M , ρ) is a complete metric space and A is a closed subset of M , prove that (A , ρ) is also complete.
15. Let $(M_1 ,\ \rho_1)$ be a compact metric space. If f is a continuous function from M_1 into a metric space $(M_2 ,\ \rho_2)$, prove that f is uniformly continuous on M_1 .
16. If f is a bounded function on $[a , b]$, prove that $f \in R[a,b]$ if and only if for each $\epsilon > 0$, there exists a subdivision σ of $[\ a , b\ ]$ such that $U[\, f \,;\, \sigma] < L[\, f \,;\, \sigma] + \epsilon$.
17. If $f \in R[a,b]$, $g \in R[a,b]$, prove that $f + g \in R[a,b]$, and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

SECTION – C **(2×20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) Let f and g be real valued functions . Prove that if f is continuous at a and if g is continuous at $f(a)$, then gof is continuous at a.

(b) Let (M_1, ρ_1) and (M_2, ρ_2) be metric spaces. Let $f: M_1 \to M_2$. Prove that f is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1 , whenever G is open in M_2 . (10 + 10)

19. (a) Prove that if the metric space M has the Heine Borel property, then M is compact.

(b) State and prove the Picard Fixed Point Theorem.

(10 + 10)

20. (a) State and prove the law of the mean.

(b) State and prove the second fundamental theorem of calculus. (10 + 10)

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