STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015-2016& thereafter)

SUBJECT CODE : 15MT/MC/GT34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
THIRD SEMESTER

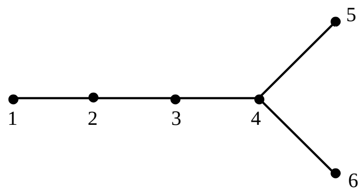
COURSE : MAJOR – CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

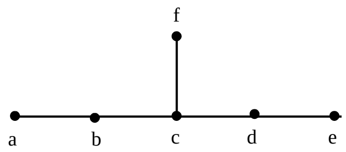
SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Prove that every cubic graph has an even number of points.
2. Define adjacency matrix.
3. Show that the partition $P = (7,6,5,4,3,2)$ is not graphic.
4. Define cutpoint of a graph.
5. Define Eulerian graph.
6. Prove that every Hamiltonian graph is 2-connected.
7. Show that $K_{3,3}$ is not planar.
8. Define crossing number.
9. Prove that every connected graph has a spanning tree.
10. Define functional digraph.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Show that the following two graphs are not isomorphic.





12. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that
    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph
    (ii) $G_1 \times G_2$ is a $(p_1p_2, q_1p_2 + q_2p_1)$ graph.
13. Prove that a graph G is connected if and only if for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2.
14. If G is a graph with $p \geq 3$ vertices and $\delta \geq {}^{p}/_{2}$, then prove that G is Hamiltonian.

15. Prove that every tree has a centre consisting of either one point or two adjacent points.
16. Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.
17. If two digraphs are isomorphic then prove that the corresponding points have the same degree pair.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) Prove that the maximum number of lines among all p points graphs with no triangle is $\left[\frac{p^2}{4}\right]$.

    (b) Prove that $c(G)$ is well defined. (12+8)

19. Let G be a connected graph with at least three points. Then prove the following statements are equivalent

    (i) G is a block.

    (ii) Any two points of G lie on a common cycle.

    (iii) Any point and any line of G lie on a common cycle.

    (iv) Any two lines of G lie on a common cycle.

20. (a) State and prove Euler's formula.

    (b) Let $G = (p, q)$ be a tree then prove that (i) every two points of G are joined by a unique path (ii) G is connected and $p = q + 1$. (10+10)