STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/ED55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ELEMENTS OF DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Write the form of a particular solution the equation $ay'' + by' + cy = ke^{ax}$, where k is a constant, when e^{ax} and xe^{ax} are solutionsof the complementary equation $ay'' + by' + cy = 0$.
2. If $y_1 = x$ is a solution of the second order differential equation $x^2y'' - 3xy' + 3y = 0$, and the general solution of $x^2y'' - 3xy' + 3y = 0$ obtained by the method of variation of parameters is $c_1x + c_2x^3$, then write the fundamental set of solutions of $x^2y'' - 3xy' + 3y = 0$.
3. If the motion of vibration string is critically damped, show that the motion is non-oscillatory.
4. Show that every point of $y'' - xy = 0$ is an ordinary point.
5. Define irregular singular point.
6. Find the the indicial polynomialof $x^2y'' - xy' - 8y = 0$.
7. Eliminate the arbitrary function from $z = f(x^2 + y^2)$.
8. Obtain the complete integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$.
9. Solve $x^2p + y^2q = z^2$.
10. Solve $\frac{\partial^2 z}{\partial x \partial y} = 0$.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Find a particular solution of $9y'' + 6y' + y = e^{-x/3}(2 - 4x + 4x^2)$.
12. If $y = \sum_{n=1}^{\infty} a_n x^n$ is a series solution of the equation $(1 + 2x^2)y'' + 10xy' + 8y = 0, y(0) = 2, y'(0) = -3$, then compute $a_{0,}a_1, a_3, a_4$.
13. Find the singular points of the Legendre's equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$. Also find their nature.
14. Find a fundamental set of Frobenius solutions of $x^2(2 - x^2)y'' - x(3 + 4x^2)y' + (2 - 2x^2)y = 0$.
15. Find a particular solution of $y'' - 3y' + 2y = \frac{4}{1+e^{-x}}$ by using variation of parameters.
16. Solve $p(1 + q^2) = q(z - 1)$.
17. By reducing the equation $z^4q^2 - z^2p = 1$ into standard form and solve it.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) Find the general solution of the equation
$(2x+1)y'' - 2y' - (2x+3)y = (2x+1)^2$, given that $y_1 = e^{-x}$ is a solution of its complementary function.

(b) Find a power series in x for the general solution of $(1+x^2)y'' + 6xy' + 6y = 0$.

19. (a) Rewrite the system $y_1{}' = 2y_1 + 4y_2 y_2{}' = 4y_1 + 2y_2$ in matrix form and verify that $y = c_1\begin{pmatrix}1\\1\end{pmatrix}e^{6t} + c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{-2t}$ satisfies the system for any choice of c_1 and c_2.

(b) Find a fundamental set of Frobenius solutions of
$x^2(3+x)y'' + 5x(1+x)y' - (1-4x)y = 0$. Also derive explicit formula for the coefficients.

20. (a) Find the complete and singular solutions of $\frac{z}{qp} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

(b) Solve $z(xp - yq) = y^2 - x^2$.