STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015–16& thereafter)

SUBJECT CODE :15MT/MC/AS55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2019
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**ALGEBRAIC STRUCTURES**
TIME	**:**	**3 HOURS** MAX. MARKS : 100

SECTION – A

Answer all questions: **(10 × 2 =20)**

1. Define abelian group. Give an example of a group which is not abelian.
2. Define cyclic group with an example.
3. If G is a finite group whose order is a prime number p, prove that G is a cyclic group.
4. Show that every subgroup of an abelian group is normal.
5. Define an automorphism of a group G.
6. Find the orbits and cycles of the permutation. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 4 & 2 & 5 & 1 & 7 & 8 & 9 \end{pmatrix}$.
7. When do you say that a commutative is an integral domain? Give an example.
8. Define a homomorphism Φ of one ring R into another R'. Specify $\Phi(0)$ and $\Phi(-a)$ for each $a \in R$.
9. If U is an ideal of $R$and$1 \in U$, prove that $U = R$.
10. Define a maximal ideal and a prime ideal.

SECTION – B

Answer any five questions: **(5 × 8 = 40)**

11. Show that $a, b \in G$, then $(a * b)^2 = a^2 * b^2$ if and only if$G$ is abelian.
12. For all a $\in$?G Prove that $Ha = \{x \in G \mid a \equiv x(mod\ H)\}$.
13. Prove that intersection of two normal subgroups of$G$ is also a normal subgroup.
14. State and prove Lagrange's theorem.
15. If G is a group, prove that the set of automorphisms of G is also a group.
16. Prove that every finite integral domain is a field.
17. If U and V are ideals of R and let $U + V = \{ u + v \mid u \in U, v \in V\}$, prove that $U + V$ is also anideal.

SECTION – C

Answer any two questions: **(2 × 20 = 40)**

18. (a) If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, prove that $o(HK) = \frac{o(H).o(K)}{o(H \cap K)}$.

    (b) Let Φ be a homomorphism if G onto $\bar{G}$ with kernel K. Prove that $G/K \approx \bar{G}$. (10+10)

19. (a) State and prove Cayley's theorem.

    (b) Let R be a Commutative Ring with unit element whose only ideals are $\{o\}$ and R itselfprove that R is a field. (10+10)

20. Prove that every integral domain can be imbedded in a field.