STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16 and thereafter)

SUBJECT CODE: 15MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

Answer All the questions **(10 x 2 = 20)**

1. Find a unit normal vector to the surface $x = t, y = t^2, z = t^3$ at $t = 1$.
2. If $\phi = r^2 e^{-r}$. Show that $grad\ \phi = (2 - r) e^{-r} \vec{r}$
3. Prove that $curl\ grad\ \phi = 0$.
4. Prove that $A \times \frac{d^2 B}{dt^2} - \frac{d^2 A}{dt^2} \times B = \frac{d}{dt}\left(A \times \frac{dB}{dt} - \frac{dA}{dt} \times B \right)$.
5. If $\vec{r} = \vec{a} Cos\, \omega t + \vec{b} Sin \omega t$. show that $\vec{r} \times \frac{d\vec{r}}{dt} = \omega (\vec{a} \times \vec{b})$ and $\frac{d^2 \vec{r}}{dt^2} = -\omega^2 \vec{r}$
6. If $\phi\ and\ \Psi$ are scalar point functions then prove that $\nabla\left(\frac{\phi}{\Psi}\right) = \frac{\Psi\ \nabla\phi - \phi\ \nabla\Psi}{\Psi^2}$.
7. Describe the coordinate surfaces and coordinate curves for cylindrical coordinates.
8. Define Osculating plane.
9. State Green’s theorem.
10. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (xdy - ydx)$.

SECTION-B

Answer any FIVE questions **(5 x 8 = 40)**

11. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$. Prove that

    i) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

    ii) $\nabla f(r) = f'(r) \frac{\vec{r}}{r} = f'(r) \nabla r$.

12. A field $\vec{F}$ is of the form $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$. Show that $\vec{F}$ is conservative field and find a function ϕ such that $\vec{F} = \nabla\phi$.

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13. Find $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ and C is the square bounded by the coordinates axes and the lines $x = a$ and $= a$.

14. Find the constants a, b and c so that
$\vec{V} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$ is irrotational. Also find $\nabla \cdot \vec{V}$.

15. Verify Stoke's theorem when $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ and surface S is the part of the sphere $x^2 + y^2 + z^2 = 1$ along the xy plane.

16. Evaluate $\iint_S \left(y^2 z\vec{i} + z^2 x\vec{j} + x^2 y\vec{k} \right) d\vec{S}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ lying in the positive octant.

17. Find $\iiint_V \vec{F}\, dv$ where $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$ and V is the volume of the region enclosed by the cylinder $x^2 + y^2 = a^2$ between the planes $z = 0$, $z = c$.

SECTION-C

Answer any TWO questions (2 x 20 = 40)

18. a) Find the equation of the tangent plane and normal to the surface $xyz = 4$ at the point (1,2,2).

b) If $\vec{u}$ and $\vec{v}$ be vector point functions. Then prove that
$\nabla \times (\vec{u} \times \vec{v}) = (\vec{v}.\nabla)\vec{u} - (\nabla.\vec{u})\vec{v} - [(\vec{u}.\nabla)\vec{v} - (\nabla.\vec{v})\vec{u}]$

19. a) Evaluate $\iint_S \vec{F}\cdot\vec{n}\, ds$ where $\vec{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first quadrant. .

b) If $\vec{A} = 2yz\,\hat{i} - x^2 y\hat{j} + xz^2\hat{k}$ and $\phi = 2x^2 yz^3$ then find (i) $(\vec{A}\cdot\nabla)\phi$ and (ii) $(\vec{A}\times\nabla)\phi$.

20. a) Verify divergence theorem for the function $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.

b) Verify Stokes theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2 z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C its boundary.

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