STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/RA55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**PRINCIPLES OF REAL ANALYSIS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ALL THE QUESTIONS

1. Using the precise definition of the limit of a function, prove that $\lim_{x\to 3}(x^2+2x)=5$.
2. Define open ball formulation of the continuity of a function $f:\mathbb{R}^1\to\mathbb{R}^1$ at a point a of $\mathbb{R}^1$.
3. Define a metric space and give an example.
4. Consider the set $\mathbb{R}^2$ with the usual metric. Is the set $A=\{(x,y)\ /\ 0\le x^2+y^2<1\}$ closed or open in $\mathbb{R}^2$. Justify your answer.
5. Define bounded sets in a metric space $<M,\rho>$. Is the set $(0,\infty)$ a bounded subset of R_d ?
6. Prove that the mapping $T:\mathbb{R}^1\to\mathbb{R}^1$ defined by $Tx=sinx$ is a contraction mapping with respect to the usual metric on $\mathbb{R}^1$.
7. State Heine-Borel property. Does the space (0,1) possess Heine-Borel property?
8. Let $<M_1,\rho_1>$, $<M_2,\rho_2>$ be two metric spaces. When do you say a function $f:M_1\to M_2$ is uniformly continuous. Is the function $f(x)=x^3,\ 0\le x\le 1$ uniformly continuous? State reasons for your answer.
9. Let $f(x)=x,(0\le x\le 1)$. Let σ be the subdivision $\{0,\frac{1}{3},\frac{2}{3},1\}$ of [0,1]. Compute $U[f,\sigma]$.
10. State the Darboux property of a function f on $[a,b]$.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

11. Prove that $\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$ does not exist.
12. Let f and g be real valued functions. If f is continuous at $a\in\mathbb{R}^1$ and g is continuous at $f(a)$, then prove that $f\circ g$ is continuous at a.

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13. Prove that l^{∞}, the set of all bounded real sequences is a metric space with respect to a suitably defined metric.
14. Prove that the arbitrary intersection of closed subsets of a metric space is closed.
15. Prove that the continuous image of a connected metric space is connected.
16. Prove that the continuous function on a compact metric space is uniformly continuous.
17. For $f(x) = sinx, \left(0 \le x \le \frac{\pi}{2}\right)$ and $\sigma_n = \{0, \frac{\pi}{2n}, \frac{2\pi}{2n}, \dots, \frac{n\pi}{2n}\}$, then prove that $\lim_{n \to \infty} U[f; \sigma_n] = 1$.

SECTION – C **(2×20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) If f is continuous at a and $f(a) > 0$, prove that there exist $h > 0$ such that $f(x) > 0, (a - h < x < a + h)$.

    (b) Let $< M, \rho >$ be a metric space. Prove that every convergent sequence of points in M is a Cauchy sequence.

    (c) Prove that arbitrary union of open subsets of a metric space $< M, \rho >$ is open.

19. (a) Prove that the subset A of $\mathbb{R}^1$ is connected if and only if whenever $a \in A, b \in A$ with $a < b$, then $(a, b) \subset A$.

    (b) Prove that the totally bounded subset of a metric space $< M, \rho >$ is bounded.

    (c) Prove that if the metric space, $< M, \rho >$ has the Heine-Borel property, then M is compact.

20. (a) If $f \in \Re[a, b]$, g $\in \Re[a, b]$, then prove that $f + g \in \Re[a, b]$ and $\int_a^b (f + g) = \int_a^b f + \int_a^b g$.

    (b) State and prove fundamental theorem of calculus.