

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 3 HOURS

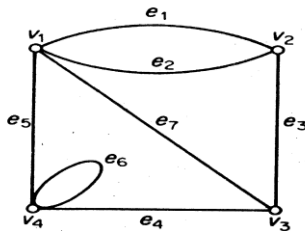
MAX. MARKS : 100

SECTION – A

(10X2=20)

ANSWER ALL THE QUESTIONS

1. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$ for (p, q) graph.
2. Find the incidence matrix for the following graph.



3. Check whether the partition $P = (7, 6, 5, 4, 3, 2)$ is graphical or not.
4. Define cut point and bridge of a graph.
5. For what values of m and n is $K_{m,n}$ eulerian?
6. Prove that every hamiltonian graph is 2-connected.
7. Prove that $K_{3,3} - e$ is planar for every edge e .
8. Define crossing number of a graph.
9. Draw all trees with 6 vertices.
10. When is a digraph said to be strongly connected?

SECTION – B

(5X8=40)

ANSWER ANY FIVE QUESTIONS

11. (a) Prove that any self complementary graph has $4n$ or $4n + 1$ points. (4)
(b) Define composition of two graphs with a suitable example. (4)
12. Prove that a closed walk of odd length contains a cycle.
13. Let G be a connected graph with at least three points. Prove that if G is a block, then any two points of G lie on a common cycle.

14. Let G be a connected graph.
- (a) Prove that if G is eulerian, then every point of G has even degree.
 - (b) Prove that if the set of edges of G can be partitioned into cycles, then G is eulerian.
15. Prove that K_5 is non-planar.
16. Prove that every tree has a centre consisting of either one point or two adjacent points.
17. Prove that the $(i, j)^{th}$ entry of A^n is the number of walks of length n from v_i to v_j .

SECTION – C**(2X20=40)****ANSWER ANY TWO QUESTIONS**

18. Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4} \right]$.
19. (a) Prove that a graph G with at least two points is bipartite if and only if all its cycles are of even length.
- (b) If G is a graph with $p \geq 3$ vertices and $\delta \geq p/2$, Prove that G is hamiltonian.
20. (a) State and prove Euler's theorem for a connected planar graph.
- (b) Prove that a weak digraph D is eulerian if and only if every point of D has equal indegree and outdegree.

