STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/MC/ED55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**ELEMENTS OF DIFFERENTIAL EQUATIONS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Find the particular solution of $y'' - 7y' + 12y = 4e^{2x}$.
2. Find the particular solution of $y'' - 2y' + y = 5cos2x + 10sin2x.$
3. Suppose a 64lb weight stretches a spring 6 inches in equilibrium and a dashpot provides a damping force of c lb for each ft/sec of velocity. Write the equation of motion of the object and determine the value of c for which the motion is critically damped.
4. Define ordinary point and singular point.
5. Find the general solution of $x^2y'' - xy' - 8y = 0 \; on \; (0, \infty)$.
6. Write the system $y_1^1 = y_1 + 2y_2 + 2e^{4t}$ and $y_2^1 = 2y_1 + y_2 + e^{4t}$ in a matrix form and also write it in initial value problem and prove that it has a unique solution on $(-\infty, \infty)$.
7. From a partial differential equation by eliminating a, b from $z = (x + a)(y + b)$.
8. Solve $pe^y = qe^x$.
9. Solve $yzp + zxq = xy$.
10. Solve $\left(D^3 - 6D^2D' + 11DD'^2 - 6D'^2\right)z = 0$.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Find a particular solution of $y'' - 4y' + 3y = e^{3x}(6 + 8x + 12x^2)$.
12. Compute $a_0 \; to \; a_7$ in the series solution $y = \sum_{n=0}^{\infty} a_n x^n$ of the initial value problem $(1 + 2x^2)y'' + 10xy' + 8y = 0, y(0) = 2, y'(0) = -3.$
13. (i) Verify that $y = \frac{1}{5}\begin{pmatrix}8\\7\end{pmatrix}e^{4t} + c_1\begin{pmatrix}1\\1\end{pmatrix}e^{3t} + c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$ is the solution of $y' = \begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}y + \begin{pmatrix}2\\1\end{pmatrix}e^{4t}$.

    (ii) Find the solution of the initial value problem $y' = \begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}y + \begin{pmatrix}2\\1\end{pmatrix}e^{4t},\; y(0) = \frac{1}{5}\begin{pmatrix}3\\22\end{pmatrix}$.
14. Solve $z = px + qy + c\sqrt{1 + x^2 + y^2}$.
15. Solve $\left(D^2 - 6DD' + 9D'^2\right)z = 12x^2 + 36xy.$
16. Solve $p(1 + q^2) = q(z - a)$.
17. Find the general solution and a fundamental Set of solutions of $x^2y'' - 3xy' + 3y = 0$ given that $y_1 = x$ is a solution.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) Solve the initial value problem $(x^2-1)y''+4xy'+2y=\frac{2}{x+1}$, $y(0)=-1, y'(0)=-5$ given that $y_1=\frac{1}{x-1}$ and $y_2=\frac{1}{x+1}$ are solutions of the complementary equation $(x^2-1)y''+4xy'+2y=0$.

(b) Let x_0 be an arbitrary real number. Find the power series in $x-x_0$ for the general solution $y''+y=0$.

19. (a) Find a fundamental set of Frobenius solution of

$$2x^2(1+x+x^2)y''+x(9+11x+11x^2)y'+(6+10x+7x^2)y=0.$$

Compute the first six coefficients $a_0\ to\ a_5$ in each solution.

(b) State Euler's equation.

(c) State regular singular point.

(d) State the existence of solution of initial value problem for linear system of differential equations.

20. (a) Solve $px+qy=z(1+pq)^{1/2}$.

(b) Solve $\frac{\partial^3 z}{\partial x^3}-\frac{\partial^3 z}{\partial y^3}=x^3y^3$.