STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16& thereafter)

SUBJECT CODE : 15MT/MC/DC14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : DIFFERENTIAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Find y_n if $y = (ax+b)^m$.
2. If $x = t^3 + 1$ and $y = t^2 - 2$ find $\frac{d^2 y}{dx^2}$.
3. Find the n^{th} differential coefficient of xe^x.
4. Define envelope of the family of curves.
5. Find the envelope of the family of circles $(x-a)^2 + y^2 = 2a$, where a is the parameter.
6. Find the radius of curvature of the curve $y = e^x$ at $x = 0$.
7. Define curvature of a curve.
8. Write the formula of radius of curvature in polar form.
9. If $u = x^3 y^2 (6 - x - y)$ then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
10. Discuss the symmetry of the following curves (a) $x^2 = y^2 \frac{y+a}{y-a}$ (b) $x^3 + y^3 = 3axy$

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Find the n^{th} differential coefficient of (a) $\frac{1}{4x^2 - 1}$ (b) $\sin x \sin 2x \sin 3x$. **(4+4)**
12. If $y = a\cos(\log x) + b\sin(\log x)$ then show that $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$.
13. Find the envelope of the family of straight lines $y + tx = 2at + at^3$, the parameter being *t*.
14. Prove that the circle of curvature of the curve $y = 4x$ at the point $(t^2, 2t)$, cuts the curve again at a point whose ordinate is -6*t* .
15. Prove that the radius of curvature of the cardioid $r = a(1 - \cos\theta)$ is $\frac{4a}{3}\sin\frac{\theta}{2}$
16. Find the *p-r* equation of the parabola $\frac{2a}{r} = 1 - \cos\theta$.
17. Show that the maximum value of $x^2 y^2 z^2$ subject to the restriction $x^2 + y^2 + z^2 = a^2$ is $\left(\frac{a^2}{3}\right)^3$.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. (a) Find the n^{th} derivative of $\sin^3 x \cos^2 x$

(b) If $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$ then prove that $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$. (10+10)

19. (a) Find the envelope of the circles drawn on the radius vectors of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter.

(b) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$. (10+10)

20. (a) Prove that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$.

(b) Trace the curve $y = (x-1)(x-2)(x-3)$. (10+10)