STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015–16 & thereafter)

SUBJECT CODE : 15MT/MC/AS55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all questions: **(10 × 2 =20)**

1. Let S be the set of all integers and let $a \sim b$ if $a - b$ is an even integer. Prove that $\sim$ is an equivalence relation of S.
2. Let G be any Group and let a$\in G$. Define the Cyclic subgroup generated by 'a'.
3. State the Lagrange's theorem on order of a subgroup.
4. Define a normal subgroup.
5. What do you mean by an automorphism of a Group G ?
6. State the Cayley's theorem.
7. Define a ring with unit element.
8. Define a ring homomorphism.
9. What do you mean by a 'Maximal Ideal' of a ring ?
10. When can you say that a ring R can be embedded in a ring $'$?

SECTION – B

Answer any five questions: **(5 × 8 = 40)**

11. Let G be a Group and let $a \in G$. Prove that $Ha = \{x \in G \ / \ a \equiv x \ mod \ H\}$.
12. If H and K are subgroups of G then prove that HK is a subgroup of G if and only if $HK = KH$.
13. If ϕ is a homomorphism of G into G' with kernel K, then prove that K is a Normal subgroup of G.
14. Let G be a Group and let ϕ be an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then prove that $o(\phi(a)) = o(a)$.
15. Prove that a finite Integral domain is a Field.

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16. If ϕ is a homomorphism of R into R', prove that $\phi(0) = 0$ and for all $a \in R,\ \phi(-a) = -\phi(a)$.
17. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a Field.

SECTION – C

Answer any two questions: **(2 × 20 = 40)**

18. (a) Prove that a non-empty subset H of the group G is a subgroup of G if and only if (i) $a, b \in H \Rightarrow ab \in H$ and (ii) $a \in H \Rightarrow a^{-1} \in H$ (5)

    (b) Let ϕ be a homomorphism of a group G onto another group G' with kernel K. Then prove that $G/K \approx G'$. (15)

19. (a) Prove that a subgroup N of G is a normal subgroup of G if and only if $gng^{-1} = N$, for every $g \in G$. (8)

    (b) Prove that S_n has a normal subgroup of index 2 the alternating group A_n consisting of all even permutations. (12)

20. (a) If 'p' is a prime number then prove that J_p , the ring of integers mod p, is a field. (5)

    (b) If ϕ is a homomrphism of R into R' with kernel $I(\phi)$ then prove that if $a \in I(\phi)$ and $r \in R$ then both 'ar' and 'ra' are in $I(\phi)$. (5)

    (c) If R is a commutative ring with unit element and M is an Ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a Field. (10)

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