STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015-2016 and thereafter)

SUBJECT CODE : 15MT/AC/ST35

B. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ALLIED – CORE
PAPER : MATHEMATICAL STATISTICS - I
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Define Sample space and Sample point.
2. A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently?
3. Define distribution function and state its properties.
4. If X and Y are two random variables having joint density function:
$$f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y); & 0 \le x < 2, 2 \le y < 4 \\ 0 & Otherwise \end{cases}$$
Find $P(X + Y < 3)$.
5. If X is a continuous random variable and 'a' is constant, then show that $E[a\,\varphi(X)] = a\,E[\varphi(X)]$.
6. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the probability that neither of them is white. Find also the probability of getting one white and one red ball. Compute the expected number of white balls drawn.
7. Define Poisson distribution and give an example.
8. Determine the mode of a binomial distribution for which the mean is 4 and variance 3.
9. Write any two characteristic of normal distribution.
10. Mention the relationship of mean, median and mode in normal distribution.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. State and prove multiplication theorem of probability.
12. A random variable X has the following probability distribution:

Value of X:	0	1	2	3	4	5	6	7	8
P(x):	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k.

(ii) Find $P(X < 3)$, $P(X \ge 3)$, $P(0 < X < 5)$.

(iii) Find the distribution function of X?

13. Let X and Y be two random variables having finite means. Prove the following:
    (a) E[Min (X,Y)] ≤ Min [E(X), E(Y)]
    (b) E[Max (X,Y)] ≥ Max [E(X), E(Y)]
14. The mean and variance of binomial distribution are 4 and 4/3. Find P(X ≥1).
15. Derive the Poisson distribution is the limiting case of Binomial distribution.
16. Derive the MGF of Normal distribution and also find mean and variance.
17. An integer is chosen at random from two hundred digits. What is the probability that the integer is divisible by 6 or 8?

SECTION – C (2X20=40)
ANSWER ANY TWO QUESTIONS

18. (a) The contents of Urns I, II and III are as follows:
    1 white, 2 black and 3 red balls,
    2 white, 1 black and 1 red balls, and
    4 white, 5 black and 3 red balls.
    One urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the probability that they come from urns I, II and III?

    (b) State and prove Chebychev's Inequality.

19. Two random variables X and Y have the following joint probability density function:

$$f(x,y) = \begin{cases} 2 - x - y; & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Find (i) Marginal probability density functions of X and Y;
(ii) Conditional density functions of X and Y;
(iii) Var (X) and Var (Y); and
(iv) Covariance between X and Y.

20. (a) Fit a Poisson distribution to the following data

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Number of pages	109	65	22	3	1

(b) The local authorities in a certain city install 10,000 electric lamps in the street of the city. If these lamps have an average life of 1,000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail (i) in the first 800 burning hours? (ii). between 800 and 1200 burning hours?

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