

B.C.A. DEGREE EXAMINATION, NOVEMBER 2018  
THIRD SEMESTER

COURSE : ALLIED – CORE  
PAPER : MATHEMATICS FOR COMPUTER SCIENCE - I  
TIME : 3 HOURS MAX. MARKS : 100

SECTION – A (10 X 2 = 20)  
ANSWER ALL THE QUESTIONS

1. Show that  $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$ .
2. Obtain the Principal disjunctive normal form for  $\neg PVQ$ .
3. Define sublattice.
4. Define lattice homomorphisms.
5. Define Mobius function  $\mu(n)$ .
6. Define divisibility.
7. Define cryptography.
8. Decipher the message FQOCUDEM using a shift transformation on single letters of 26 letter alphabet, where E is the most frequently occurring letter in English language.
9. There are 15 married couples in a party. Find the number of ways of choosing a woman and a man such that the two are (a) married to each other (b) not married to each other.
10. What is pigeonhole principle?

SECTION – B (5 X 8 = 40)  
ANSWER ANY FIVE QUESTIONS

11. Show that  
(a)  $\neg(P \rightarrow Q) \Leftrightarrow (PVQ) \wedge \neg(P \wedge Q)$   
(b)  $\neg(P \rightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$
12. Obtain the conjunctive normal form of the formula  
 $(p \wedge \neg(q \vee r)) \vee (((p \wedge q) \vee \neg r) \wedge p)$
13. Prove that in any lattice  $(L, \leq)$ , the operations  $\vee$  and  $\wedge$  are isotone. i.e., if  $y \leq z$  is in  $L$ , then  $x \wedge y \leq x \wedge z$  and  $x \vee y \leq x \vee z$ , for all  $x \in L$ .
14. Prove that every integer  $n > 1$  is either a prime number or product of prime numbers.
15. Solve  $2x + 3y \equiv 1 \pmod{26}$   
 $7x + 8y \equiv 2 \pmod{26}$
16. A committee of 7 members is to be chosen from 6 chartered accountants, 4 Economists and 5 Cost Accountants. In how many ways can this be done if in the committee there must have atleast one member from each group and atleast 3 Chartered Accountants.
17. Show that in any group of 6 people there will always be a subgroup of 3 people who are pairwise acquainted or a subgroup of 3 people who are pairwise strangers

**SECTION – C**  
**ANSWER ANY TWO QUESTIONS**

(2 X 20 = 40)

18. a) Show that  $(PVQ) \wedge \neg (\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$  is a tautology.
- b) If  $(L, \wedge, \vee)$  and  $(M, \sqcap, \sqcup)$  are lattices then prove that  $(L \times M, \wedge, \vee)$  is a lattice.
19. a) A chest contains 20 shirts, of which 4 are tan, 7 are white and 9 are blue. At the least how many shirts must one remove (blindfolded) to get  $r = 4, 5, 6, 7, 8, 9$  shirts of the same colour.
- b) Suppose that we know that our adversary is using a  $2 \times 2$  enciphering matrix with a 29 – letter alphabet, where A-Z have the usual numerical equivalents, blank = 26, ? = 27, ! = 28. We receive the message.
- GFPYJP X? UYXSTLADPLW
- and we suppose that we know that the last five letters of plaintext are adversary's signature KARLA. Find the deciphering matrix and find the full plaintext message.
20. a) State and prove the fundamental theorem of arithmetic
- b) Prove that given integers  $a$  and  $b$  with  $b > 0$ , there exists a unique pair of integers  $q$  and  $r$  such that  $a = bq + r$ , with  $0 \leq r < b$ . Moreover  $r = 0$  iff  $b|a$ .

