STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/PE/FT14

M. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : ELECTIVE
PAPER : FUZZY SET THEORY AND APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. Define a normal and a sub normal fuzzy set.
2. Explain crisp and binary relation.
3. Give an example of a fuzzy complement that is continuous but not involute.
4. Define a fuzzy number.
5. Mention any two instances where fuzzy logic is used in our day today home appliances.

SECTION – B

Answer any five questions: **5×6=30**

6. Prove that a fuzzy set A on R is convex if and only if
   $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$.
7. Let P and Q be two binary relations given by the following membership matrices

$$P = \begin{pmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & .0.6 & 0.5 \end{pmatrix} \text{ and } Q = \begin{pmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{pmatrix}.$$

   Find the standard composition $P \circ Q$.
8. Prove that every fuzzy complement has at most one equilibrium.
9. Let A and B be two triangular fuzzy numbers, given by

$$A(x) = \begin{Bmatrix} 0 \;\; for \;\; x \leq -1 \;\; and \;\; x > 3 \\ \dfrac{(x+1)}{2} \;\; for \;\; - < x \leq 1 \\ \dfrac{(3-x)}{2} \;\; for \;\; 1 < x \leq 3. \end{Bmatrix} \quad and \quad B = \begin{Bmatrix} 0 \;\; for \;\; x \leq 1 \;\; and \;\; x > 5 \\ \dfrac{(x-1)}{2} \;\; or \;\; 1 < x \leq 3 \\ \dfrac{(5-x)}{2} \;\; for \;\; 3 < x \leq 5. \end{Bmatrix}.$$

   Find $A + B$ and $A - B$.

10. Explain the methods of designing fuzzy controller.

11. For any $A \in F(X)$, prove that $^{\alpha}A = \bigcap_{\beta<\alpha} {}^{\beta}A = \bigcap_{\beta<\alpha} {}^{\beta+}A$.

12. Discuss applications of fuzzy logic to medicine.

SECTION – C

Answer any three questions: **3×20=60**

13. (a) Explain why the law of contradiction and law of excluded middle are violated in fuzzy set theory under standard fuzzy set operations..
    (b) Write down the features responsible for the Paradigm shift from the classical set theory. (10 + 10)

14. Let $f : X \rightarrow Y$ be an arbitrary crisp function. Then for any $A \in F(X)$ and $\alpha \in [0, 1]$, prove that
    $(i).\ {}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$
    $(ii).\ {}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$.
    Show further that ${}^{\alpha}[f(A)] \neq f({}^{\alpha}A)$ in general. If X is finite does the equality hold in (ii).
    (10 + 10)

15. (a) Prove that $u(a, b) = \max(a, b)$ is the only continues and idempotent fuzzy set union.
    (b) Prove that $i(a, b) = \min(a, b)$ is the only continues and idempotent fuzzy set intersection.
    (10 +10)

16. (a) Find the solution of the fuzzy equation $A.X = B$, where A and B are fuzzy numbers on R^{+} given as follows.

$$A(x) = \begin{cases} 0 & for\ x \le 3\, and\ x > 5 \\ x - 3 & for\ 3 < x \le 4 \\ 5 - x & for\ 4 < x \le 5 \end{cases}$$

$$B(x) = \begin{cases} 0 & for\ x \le 12\, and\ x > 32 \\ (x-12)/8 & for\ 12 < x \le 20 \\ (32-x)/12 & for\ 20 < x \le 32 \end{cases}$$

   (b) Determine which fuzzy sets defined by the following functions are fuzzy numbers?

$$(i).\ A(x) = \begin{cases} x & for\quad 0 \le x \le 1 \\ 0 & otherwise. \end{cases}$$

$$(ii).\ B(x). = \begin{cases} \min(1, x) & for\quad x \ge 0 \\ 0 & for\quad x < 0. \end{cases}$$

(10 +10)

17. Discuss any one application of fuzzy mathematics in industry.