STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/PC/MI24

M. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : MEASURE THEORY AND INTEGRATION
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. Define measureable sets.
2. Define rings and give an example.
3. Prove that every continuous function is measurable.
4. Let $f = g$ a.e. (μ), where μ is a complete measure. Show that if f is measurable, so is g.
5. Show that if f is a non-negative measurable function then $\int f\, dx = 0$ if and only if , $f = 0$ $a.e.$
6. When do you say a bounded function f on $[a, b]$ is Riemann integrable?
7. Define convex and strictly convex functions.
8. If $f_n \to f$ in the mean of order p $(p > 0)$, prove that $f_n \to f$ in measure.
9. Define signed measures.
10. Define Radon-Nikodym derivative.

SECTION – B

Answer any five questions: **5×6=30**

11. For any sequence of sets $\{E_i\}$, prove that $m^*(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(E_i)$.
12. Prove that every interval is measurable.
13. If f and g are measurable functions on the same measurable set E prove that $f + g$, $f - g$ and fg are measurable.
14. Let $f(x), (0 \leq x \leq 1)$ be defined by $f(x) = \begin{cases} 0 & if\ x\ is\ rational \\ n & if\ x\ is\ irrational \end{cases}$

    where n is the number of zeros immediately after the decimal representation of x.

    Show that f is measurable and compute $\int_0^1 f\, dx$.

15. If f and g be non-negative measurable functions then prove that

$$\int f\,dx + \int g\,dx = \int (f+g)dx.$$

16. Prove that every convex function on an open interval is continuous.

17. If $\int f\,d\mu$ is defined and if $\varphi(E) = \int_E f\,d\mu$ prove that φ is a signed measure.

SECTION – C

Answer any three questions: 3×20=60

18. a) Prove that the class $\mathfrak{M}$ of all Lebesque measurable sets is a σ −algebra.

    b) If $m^*(E) < \infty$ then prove that E is measurable if and only if $\forall\varepsilon > 0$, there exists disjoint finite intervals $I_1, I_2, \ldots, I_n$ such that $m^*(E\Delta \bigcup_{i=1}^{n} I_i) < \varepsilon$.

19. a) Establish the existence of non-measurable sets.

    b) Prove that not every measurable set is a Borel set.

20. State and prove :

    a) Fatou's Lemma.

    b) Dominated convergence theorem.

21. a) If $1 \le p \le \infty$ and $\{f_n\}$ is a sequence in $L^p(\mu)$ such that $\|f_n - f_m\| \to 0$ as $n, m \to \infty$ prove that there exists a function f and a subsequence $\{n_i\}$ such that $\{f_{n_i}\} = 0\ a.e,$ $f \in L^p(\mu)$ and $lim \quad \|f_n - f_m\|_p = 0.$

    b) If $\{f_n\}$ is a sequence of measurable functions which is fundamental in measure, prove that there exists a measurable function f such that $f_n \to f$ in measure.

22. State and prove Radon-Nikodym theorem.