STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16 & thereafter)

SUBJECT CODE: 15MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION - A (5x2=10)
ANSWER ALL THE QUESTIONS

1. State the Jensen's inequality.
2. Define "Eigen spectrum"
3. What do you mean by NORMED DUAL of a space X?
4. State the parallelogram law for inner products on a linear space X.
5. Define "Normal" and "Self Adjoint" operators.

SECTION - B (5x6=30)
ANSWER ANY FIVE QUESTIONS

6. Let X be a normed space over K and f be a non zero linear functional on X. If E is an open subset of X then prove that $f(E)$ is an open subset of K.
7. Let X and Y be Banach spaces: $F: X \rightarrow Y$ be a closed linear map. Then prove that F is continuous.
8. Let X be a separable normed space. Then prove that every bounded sequence in X' has a weak* convergent subsequence.
9. Let X be a normed space and let Y be a Banach space and let X_o be a dense subspace of X and $F_0 \in BL\,(X_0, Y)$ then prove that there is a unique $F \in BL\,(X, Y)$ such that $F_{|X_0} = F_0$.
10. State and prove the schwarz's inequality for inner products on a linear space X.
11. Let H be an Hilbert space and $A, B \in BL(H)$ If A and B are normal such that A commutes with B^* and B commutes with A^* then prove that $A + B$ and AB are normal.
12. Let H be a Hilbert space. Let A and B be self adjoint. Then prove that $A + B$ is self adjoint. Further prove that AB is self adjoint if and only if A and B commute with each other.

SECTION - C (3x20=60)
ANSWER ANY THREE QUESTIONS

13. Let X be a normed space. Then prove that the following statements are equivalent.

(i) Every closed and bounded subset of X is compact.

(ii) The subset $\{ x \in X : \ || x || \leq 1\}$ of X is compact

(iii) X is finite dimensional. 20 Marks

14. State and prove Hahn - Banach extension theorem. 20 Marks

15. a) Let X be a normed space and let E be a subset of X. Then prove that E is bounded in X if and only if $f(E)$ is bounded in K for every $f \in X'$. Where X' is dual of normed space X. 10 Marks

b) Let X and Y be normed spaces and let $F: X \rightarrow Y$ be linear then prove that F is continues if and only if goF is continuous for every $g \in Y'$. Where Y' is dual of normed space Y ? 10 Marks

16. a) State and prove the Bessel's Inequality?

b) State and prove Riesz - Fisher theorem for countable orthonormal set in an inner product space.

17. Let H be a Hilbert space and let $A \in BL\,(H)$

a) Let A be self adjoint then prove that $||A|| = sup\,\{| < A(x),\ x > |: x \in H, ||x|| \leq 1\}$ further prove that $A = 0$ if and only if $< A(x), x > = 0$ for all $x \in H$. 10 Marks

b) A is unitary if and only if $\left||A(x)|\right| = ||x||$ for all $x \in X$ and if A is surjective prove that $||A^{-1}\,(x)\,|| \ = \ ||x||$ for all $x \in H$ and $||A|| = 1 = \ ||A^{-1}||$. 10 Marks