STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16 & thereafter)

SUBJECT CODE: 15MT/PC/DG44

M. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**DIFFERENTIAL GEOMETRY**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION - A (5x2=10)
ANSWER ALL THE QUESTIONS

1. Compute the curvature of $\gamma(t) = (t, cosht)$.
2. Define loxodrome curve.
3. Compute the first fundamental form of the surface $\sigma(u, v) = (coshu, sinhu, v)$.
4. When do you say a curve γ on a surface S is asymptotic?
5. Prove that any normal section of a surface is a geodesic.

SECTION - B (5x6=30)
ANSWER ANY FIVE QUESTIONS

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6. Find the curvature of the circular helix $\gamma(\theta) = (acos\theta, asin\theta, b\theta), -\infty < \theta < \infty$, where a and b are constants.
7. Let $f: S_1 \to S_2$ be a diffeomorphism. If σ_1 is an allowable surface patch on S_1, prove that $f \circ \sigma_1$ is an allowable surface patch on S_2 .
8. Prove that any tangent developable is isometric to a plane.
9. State and prove Meusnier's theorem.
10. Prove that a curve on a surface is geodesic if and only if its geodesic curvature is zero everywhere.
11. Prove that the area of a surface patch is unchanged by reparametrisation.
12. Prove that an isometry between two surfaces take the geodesics of one surface to the geodesic of the other.

SECTION - C (3x20=60)
ANSWER ANY THREE QUESTIONS

13. (a) Prove that a parametrised curve has a unit speed reparametrisation if and only if it is regular.

(b) Let $\bar{\gamma}(t)$ be a regular curve in R^3 with nowhere vanishing curvature. Prove that denoting d/dt by a dot, its torsion is given by $\tau = \frac{(\dot{\bar{\gamma}} X \ddot{\bar{\gamma}}).\dddot{\bar{\gamma}}}{||\dot{\bar{\gamma}} X \ddot{\bar{\gamma}}||^2}$. (10 + 10)

14. (a) Let U and $\bar{U}$ be open subsets of R^2 and let $\sigma: U \to R^3$ be a regular surface patch. Let $\emptyset: \bar{U} \to U$ be a bijective smooth map with smooth inverse map $\emptyset^{-1}: U \to \bar{U}$. Prove that $\tilde{\sigma} = \sigma \; o \; \emptyset : \bar{U} \to R^3$ is a regular surface patch.

(b) Show that the Möbius band is not orientable. (10 +10)

15. Prove that a diffeomorphism $f: S_1 \to S_2$ is conformal if and only if, for any surface patch σ_1 on S_1, the first fundamental forms of σ_1 and $f o \sigma_1$ are proportional.

16. Let κ_1 and κ_2 be the principal curvatures at a point P of a surface patch σ.
Prove that

(i) κ_1 and κ_2 are real numbers;

(ii) if $\kappa_1 = \kappa_2 = \kappa$, say then $\mathcal{F}_{II} = \kappa \mathcal{F}_I$ and (hence) every tangent vector to σ at P is a principal vector;

(iii) if $\kappa_1 \neq \kappa_2$, then any two (non-zero) principal vectors t_1 and t_2 corresponding to κ_1 and κ_2, respectively, are perpendicular.

17. State and prove Gauss's theorem along with necessary lemma.

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