STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16 & thereafter)

SUBJECT CODE: 15MT/PC/CI44

M. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : CALCULUS OF VARIATION AND INTEGRAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION—A (5x2=10)
ANSWER ALL THE QUESTIONS

1. In extremizing the functional $I[y(x)] = \int_a^b F(x, y(x), y'(x))dx$ if F depends only on x and y', then write the Euler's equation.
2. Show that the diffracted rays form the surface of the cone.
3. Find the kernel of the of the integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}} dt = \sqrt{x}$.
4. Reduce the initial value problem $y' - y = 0;\ y(0) = 0$ into Volterra integral equation of the second kind.
5. Define index of eigenvalue.

SECTION—B (5x6=30)
ANSWER ANY FIVE QUESTIONS

6. Using Bernstein theorem show that there exists an extremal through any two points with distinct abscissae for the functional $I = \int_0^x e^{-2y^2}({y'}^2 - 1)\, dx$
7. Find the extremum of the functional $\int_{x_1}^{x_2} \left({y'}^2 + {z'}^2 + 2yz\right)dx\ with\ y(0) = 0 = z(0)$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.
8. Show that $y(x) = xe^x$ is the solution of the integral equation $y(x) = \sin x + 2\int_0^x \cos(x-t)y(t)dt.$
9. Find the initial value problem corresponding to the integral equation $y(x) = 1 + \int_0^x y(t)dt$.
10. Show that the in the integral equation $y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t\ y(t)dt$ has no eigenfunctions.
11. Reduce the initial value problem $y'' - 5y' + 6\mathrm{y} = 0; \mathrm{y}(0) = 0;\ y'(0) = -1$ into Volterra integral equation of the second kind.
12. Find the shortest path from the point A(-2,3) to the point (2,3) located in the region $y \leq x^2$.

SECTION—C (3x20=60)
ANSWER ANY THREE QUESTIONS

13. (a) Determine the shape of a solid of revolution moving in a flow of gas with least resistance.
(b) Derive Euler-Poisson equation in testing the extremum of the functional depending on higher-order derivatives.

14. (a) Find the extremals with corner points of the functional
$I[y(x)] = \int_{c_1}^{x_2} y'^2(1-y')^2\, dx$.
(b) Find the curve that extremizes the functional $I[y(x)] = \int_{x_1}^{x_2} F(x,y,y')dx$ and passes through the points $A(x_1, y_1)$ and (x_2, y_2) , the curve must arrive at B only after reflection from a given curve $y = f(x)$.

15. (a) Show that $y(x) = e^x(\cos e^x - e^x \sin e^x)$ is a solution of the integral equation
$y(x) = (1 - xe^{2x})\cos 1 - e^{2x}\sin 1 + \int_0^x [1-(x-t)e^{2x}]y(t)dt$.
(b) Show that the function $y(x) = \sin(\frac{\pi x}{2})$ is a solution of the Fredholm integral equation
$y(x) - \frac{\pi^2}{4}\int_0^1 K(x,t)y(t)dt = \frac{x}{2}$, where $K(x,t) = \begin{cases} \frac{x(2-t)}{2} & if\ 0 \le x \le t \\ \frac{t(2-x)}{2} & if\ t \le x \le l \end{cases}$

16. Obtain Fredholm integral equation of second kind corresponding to the boundary value problem $\frac{d^2\varphi}{dx^2} + \lambda\varphi = x;\ \varphi(0) = 0;\ \varphi(1) = 1$. Also recover the boundary value problem from the integral equation obtained.

17. (a) Show that the homogeneous integral equation $y(x) = \lambda\int_0^1 (t\sqrt{x} - x\sqrt{t})y(t)dt$ does not have real eigenvalues and eigen functions.
(b) Find the eigenvalues and eigenfunctions of the homogeneous integral equation
$y(x) = \lambda\int_{-1}^{1}(5xt^3 + 4x^2\, t + 3xt)y(t)dt$.