STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16B& thereafter)

SUBJECT CODE : 15MT/MC/VL65

B. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS. **(10×2=20)**

1. Define isomorphism on Vector spaces.
2. If V is a vector space over F then prove that $\alpha 0 = 0$ for $\alpha \in F$.
3. If F is the field of real numbers then prove that the vectors (1, 1, 0, 0), (0, 1,-1, 0) and (0, 0, 0, 3) are linearly independent over F.
4. Define a linear combination of $v_1, v_2, ..., v_n$ over the field F.
5. Define an orthogonal complement of a subspace W of a vector space V.
6. Prove that $W^{\perp}$ is a subspace of V.
7. If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then prove that $\lambda v = vT$.
8. If $T \in A(V)$ and if $S \in A(V)$ is regular then prove that $r(T) = r(STS^{-1})$.
9. Prove that the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$ is diagonalizable.
10. Consider the linear operator $T(x, y) = (3x + y, x + 3y)$ on R^2. Find a matrix representation of T.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5×8=40)**

11. Let V_n be the set of polynomials of degree less than n. Prove that V_n is a vector space over F.
12. Prove that F^n is isomorphic to F^m if and only if $n = m$.
13. If a, b, c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ then prove that $b^2 \leq ac$.
14. If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then prove that λ is a root of the minimal polynomial of T.
15. Consider the linear transformation $T: R^3 \to R^2$ defined as $(1,0,0) = (3,-1)$, $T(0,1,0) = (2,1)$, $T(0,0,1) = (3,0)$. Find $T(1,-2,3)$.
16. If U and W are subspaces of V then prove that $U + W = \{v \in V : v = u + w, u \in U, w \in W\}$ is a subspace of V.
17. Prove that similar matrices have same eigenvalues.

SECTION –C

ANSWER ANY TWO QUESTIONS. **(2×20=40)**

18. (a) If F is the internal direct sum of $U_1, U_2, ..., U_n$ then prove that V is isomorphic to the external direct sum of $U_1, U_2, ..., U_n$.

    (b) If V is finite dimensional and if W is a subspace of V then prove that $\dim \frac{V}{W} = \dim V - \dim W$.

19. (a) Let V be with the set of all polynomials of degree ≤ 2 together with the zero polynomial. V is a inner product defined by $(p(x), q(x)) = \int_{-1}^{1} p(x)q(x)dx$, $p(x), q(x) \in V$ Starting with the basis $\{1, x, x^2\}$, obtain an orthonormal basis for V.

    (b) If V is finite dimensional over F then prove that T is invertible if and only if the constant term of the minimal polynomial for T is not 0.

20. Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \end{bmatrix}$ by orthogonally.