STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/MC/SF45

B. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE, SERIES AND FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. Define : Countable set.
2. Define : least upper bound of a set.
3. Define : limit of a sequence.
4. When do you say that a sequence is bounded?
5. When do you say that a sequence is a Cauchy sequence?
6. Define : absolute convergence of a series.
7. State the comparison test for absolute convergence of a series.
8. State : Abel's Lemma..
9. Define : odd and even function.
10. Find the Fourier coefficient a_o for $f(x) = x^2, \ -\pi \le x \le \pi.$

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. Prove that the countable union of countable sets is countable.
12. Show that if $lim_{n\to\infty} s_n = L$ and $lim_{n\to\infty} s_n = M$, then $L = M.$
13. If $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
14. Show that a convergent sequence is a Cauchy sequence.
15. If $\sum_{n=1}^{\infty} a_n$ is a series of nonnegative numbers with $s_n = a_1 + a_2 + \cdots .. + a_n (n \epsilon I),$ prove that $\sum_{n=1}^{\infty} a_n$ converges if $\{s_n\}_{n=1}^{\infty}$ is bounded.
16. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be sequences of real numbers and let $s_n = a_1 + a_2 + \cdots .. + a_n$. Prove that for each $n \epsilon I,$
    $\sum_{k=1}^{n} a_k b_k = s_n b_{n+1} - \sum_{k=1}^{n} s_k (b_{k+1} - b_k).$
17. Find a sine series for $f(x) = c$, in the range 0 to π, where c is a constant.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

18. (a) Show that the set of all real numbers is uncountable.

    (b) If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers and if $lim_{n\to\infty} s_n = L,\ lim_{n\to\infty} t_n = M$, show that $lim_{n\to\infty}(s_n + \ t_n\,) = L + M$.

    (10 + 10)

19. (a) State and prove the Ratio test.

    (b) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

    (10 + 10)

20. (a) State and prove the nested interval theorem.

    (b) Express $f(x) = \frac{1}{2}\ (\,\pi - x\,)$ as a Fourier series to be valid in the interval 0 to 2π .

    (10 + 10)