STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/MC/ML24

B. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : MULTIPLE INTEGRAL AND LAPLACE TRANSFORMS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION A

Answer All Questions: **10 x 2 = 20**

1. Evaluate $\int_1^2 \int_1^x xy^2 dydx$.
2. Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.
3. If $x = rcos\theta, y = rsin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
4. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{2acos\theta} r^3 drd\theta$.
5. Define Beta function.
6. Prove that $\Gamma(n+1) = n!$.
7. Define Laplace transform of a function.
8. Find $L(e^{-st})$.
9. If $L\big(f(t)\big) = F(s)$ then prove that $L\big(f(at)\big) = \frac{1}{a}F\left(\frac{s}{a}\right)$.
10. Find $L^{-1}\left(\frac{1}{(s+2)^2+16}\right)$.

SECTION B

Answer Any Five Questions: **5 x 8 = 40**

11. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xydydx$ and evaluate it.
12. Evaluate $\int\int r\sqrt{a^2-r^2}\ \ dr\ d\theta$ over the upper half of the circle $r = acos\theta$.
13. Evaluate $\int\int_R\ xy\ dx\ dy$, where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$ and $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 = d^2$ $(0 < a < b < c < d)$.
14. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

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15. Evaluate $\int_0^1 x^m \left(log\frac{1}{x}\right)^n dx$.

16. Find (i) $L(t^2+2t+3)$ (ii) $L(sin^2 2t)$ (iii) $L(cos\ at)$. (3+3+2)

17. Find (i) $L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right)$ (ii) $L^{-1}\left(\frac{1}{s(s+a)}\right)$. (5+3)

SECTION C

Answer Any Two Questions: **2 x 20= 40**

18. (a) Evaluate $\int\int\int xyz\ dx\ dy\ dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

(b) Evaluate $\int\int_R (x-y)^4 e^{x+y}\ dx\ dy$ where R is the square with vertices $(1,0), (2,1), (1,2)$ and $(0,1)$. (12+8)

19. (a) By transforming into polar coordinates evaluate $\int\int \frac{x^2y^2}{x^2+y^2} dx\ dy$ over the annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ $(a<b)$

(b) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function and evaluate the integral $\int_0^1 x^5(1-x^3)^{10} dx$. (10+10)

20. (a) Find the Laplace transform of the function $f(t)=\begin{cases} 1, & 0<t<b \\ -1, & b<t<2b \end{cases}$.

(b) Solve the equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=sint$, given that $y=\frac{dy}{dt}=0$ when $t=0$. (8+12)