STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16)

SUBJECT CODE : 15MT/MC/CA65

B. Sc. DEGREE EXAMINATION, APRIL 2019
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 × 2 = 20**

1. When is a complex function said to be differentiable at a point?
2. Check whether the function $3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic or not.
3. When is a mapping said to be a conformal mapping.
4. Determine the angle of rotation at the point $z = 1 + i$ under the mapping $w = z^2$.
5. Define bilinear transformation.
6. Obtain the Taylor series expansion of e^z about $z = 0$.
7. Write down Cauchy's inequality.
8. State Liouville's theorem.
9. Define isolated singularity of a function.
10. Find the zeros and order of zeros of $(z - 4i)^4 (z + 3)^5$.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 × 8 = 40**

11. Derive Cauchy Riemann equations in polar form.
12. Show that the function $f(z) = \begin{cases} \dfrac{z \operatorname{Re} z}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at $z = 0$.

    but not differentiable at z = 0.
13. Find the images of $y = c,\ x = c$ and the rectangular region bounded by

    $a \le x \le b, c \le y \le d$ under the mapping $w = e^z$.
14. Find the Taylor's series expansion of $\dfrac{z^2 - 1}{(z+2)(z+3)}$ in $|z| < 2$.

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15. State and prove Cauchy integral formula and hence evaluate $\int_C \frac{e^z}{z^2+4}\,dz$ where C is the circle $|z-i|=2$.

16. State and prove maximum modulus theorem.

17. Using Cauchy residue theorem, Evaluate $\int_C \frac{(3z-4)}{z(z-1)(z-2)}dz$ where C is the circle $|z|=\frac{3}{2}$.

SECTION-C

ANSWER ANY TWO QUESTIONS: **2 × 20 = 40**

18. (a) If $f(z)=u+iv$ is differentiable at a point $z_0=x_0+iy_0$. Prove that u and v have first order partial derivatives at (x_0,y_0) and these partial derivatives satisfy the Cauchy Riemann equations.

(b) Discuss the transformation $w=\sin z$. (10+10)

19. State and prove Laurent's theorem and obtain the Laurent's series.

expansion of $\frac{e^{2z}}{(z-1)^3}$ about $z=1$.

20. (a) State and prove Cauchy's theorem.

(b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}d\theta$ (10+10)