STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600086 (For candidates admitted from the academic year 2015-16\& thereafter)

SUBJECT CODE : 15MT/AC/MC25

## B. Sc. DEGREE EXAMINATION, APRIL 2019 <br> BRANCH IV - CHEMISTRY <br> SECOND SEMESTER

## COURSE : ALLIED CORE

PAPER : MATHEMATICS FOR CHEMISTRY - II TIME : 3 HOURS

MAX. MARKS : 100

## SECTION - A

## ANSWER ALL THE QUESTIONS:

(10X2=20)

1. Define normal subgroup.
2. If $G$ is a group such that $a^{2}=e$ for all $a \in G$, then prove that $G$ is abelian.
3. Show that $L\left(e^{-a t}\right)=\frac{1}{s+a}$ provided $s+a>0$.
4. Show that $\left[f^{\prime}(t)\right]=s L[f(t)]-f(0)$.
5. Find $L^{-1}\left(\frac{s-3}{(s-3)^{2}+4}\right)$.
6. Find $L^{-1}\left[\frac{s}{(s+2)^{2}}\right]$.
7. Define even function and odd function.
8. Express $f(x)=x(-\pi<x<\pi)$ as a Fourier series with period $2 \pi$.
9. Define correlation coefficient.
10. If $r=0.6$ and $N=64$, find out the probable error of the coefficient of correlation and determine the limits for population $r$.

## SECTION - B

## ANSWER ANY FIVE QUESTIONS:

$(5 \times 8=40)$
11. Let $G$ be a group. Let $H=\{a \mid a \in G$ and $a x=x a$ for all $x \in G\}$ (ie) $H$ is the set of all elements which commute with all other element. Show that $H$ is a subgroup of $G$.
12. Express $f(x)=\frac{1}{2}(\pi-x)$ as a Fourier series with period $2 \pi$, to be valid in the interval 0 to $2 \pi$.
13. Show that $L\left(t^{n}\right)=\frac{n!}{s^{n+1}}$ where $n$ is a positive integer.
14. Find $L\left(e^{-a t} \sin b t\right)$.
15. Find $L^{-1}\left(\frac{s-3}{s^{2}+4 s+13}\right)$.
16. A computer while calculating correlation coefficient between two variables X and Y from 25 pairs if observations obtained the following results:
$N=25, \quad \sum X=125, \quad \sum X^{2}=650, \quad \sum Y=100, \quad \sum Y^{2}=460, \quad \sum X Y=508$
It was however, later discovered at the time of checking that he had copied down two pairs as $(6,14)$ and $(8,6)$ while the correct values were $(8,12)$ and $(6,8)$. Obtain the correct value of correlation coefficient.
17. Obtain the rank correlation coefficient for the following data.

| X | 68 | 64 | 75 | 50 | 64 | 80 | 75 | 40 | 55 | 64 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 62 | 58 | 68 | 45 | 81 | 60 | 68 | 48 | 50 | 70 |

## SECTION - C

## ANSWER ANY TWO QUESTIONS:

$(2 \times 20=40)$
18. a) Let $G$ be a finite group with even number of elements. Prove that $G$ contains at least one element of order 2.
b) Evaluate $L(t \sin a t)$.
(12 Marks)
(8 Marks)
19. a) Evaluate $L^{-1}\left[\frac{1}{(s+1)\left(s^{2}+2 s+2\right)}\right]$.
(5 Marks)
b) Solve the equation $\frac{d^{2} y}{d x^{2}}+4 \frac{d y}{d x}+13 y=2 e^{-x}$, given $y=0, \frac{d y}{d x}=-1$ when $x=0$
20. a) Calculate Karl Pearson's coefficient of correlation from the following data. (10 Marks)

| Age of mother's in years | Age of daughters in years |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ |  |
| $15-25$ | 6 | 3 | - | - | - | 9 |
| $25-35$ | 3 | 16 | 10 | - | - | 29 |
| $35-45$ | - | 10 | 15 | 7 | - | 32 |
| $45-55$ | - | - | 7 | 10 | 4 | 21 |
| $55-65$ | - | - | - | 4 | 5 | 9 |
| Total | 9 | 29 | 32 | 21 | 9 | 100 |

b) Show that in $0 \leq x \leq \pi, x(\pi-x)=\frac{\pi^{2}}{6}-4\left[\frac{\cos 2 \mathrm{x}}{2^{2}}+\frac{\cos 4 \mathrm{x}}{4^{2}}+\frac{\cos 6 \mathrm{x}}{6^{2}}+\cdots \infty\right]$. Deduce the sum of the series $\frac{1}{1^{2}}-\frac{1}{2^{2}}-\frac{1}{3^{2}}-\frac{1}{4^{2}} \ldots \infty$.

